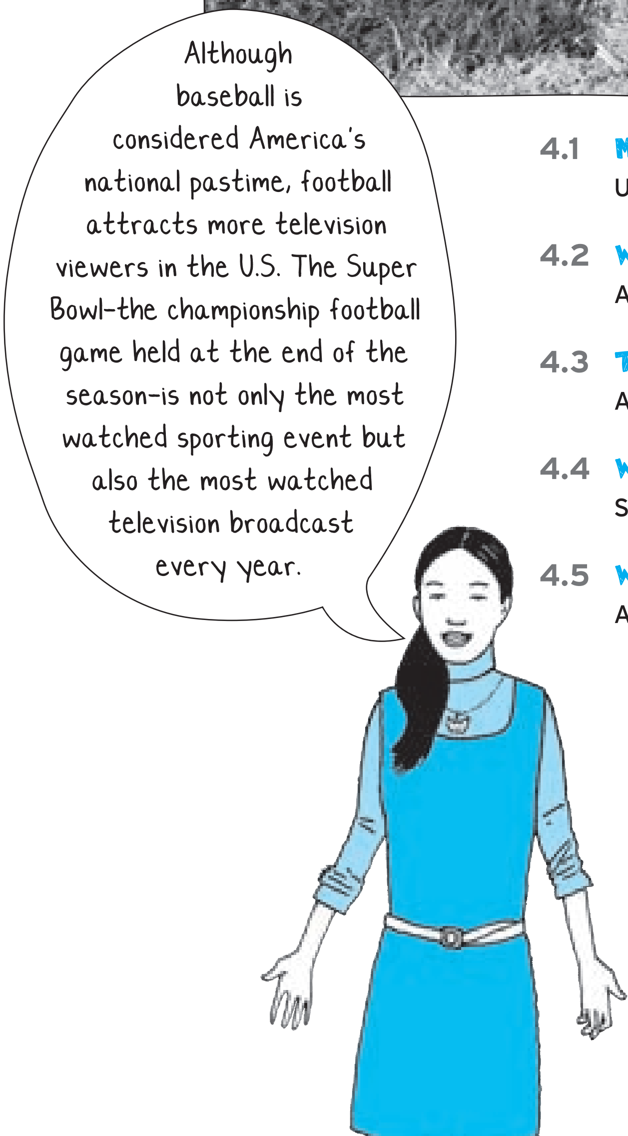
4 ADDITION AND SUBTRACTION WITH RATIONAL NUMBERS



Chapter 4 Overview

This chapter uses models to develop a conceptual understanding of addition and subtraction with respect to the set of integers. These strategies are formalized through questioning, and then extended to operations with respect to the set of rational numbers.

Lessons		CCSS	Pacing	Highlights	Models	Worked Examples	Peer Analysis	Talk the Talk	Technology
4.1	Using Models to Understand Integers	7.NS.1.a 7.NS.1.b	1	This lesson includes the game "Math Football" as a model to think about how positive and negative quantities describe direction. Questions ask students to connect the moves from the game "Math Football" to number sentences that include positive and negative integers.	X	X			
4.2	Adding Integers, Part I	7.NS.1.b	1	This lesson connects the concepts of positive and negative integers developed in "Math Football" to the number line. Questions ask students to add integers using the number line and to think about the distances and absolute values of each integer. No formal rules for adding integers are established yet, however, questions will ask students to analyze the models for patterns.	X	X			
4.3	Adding Integers, Part II	7.NS.1.a 7.NS.1.b 7.NS.1.c	1	This lesson uses two-color counters as a different model to represent the sum of two integers with particular emphasis on zero and additive inverses. Questions ask students to notice patterns and write a rule for adding integers, and then display their understanding of additive inverse and zero using words, number sentences, a number line model, and a two-color counter model in a graphic organizer.	X	X		X	

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Lessons		CCSS	Pacing	Highlights	Models	Worked Examples	Peer Analysis	Talk the Talk	Technology
4.4	Subtracting Integers	7.NS.1.a 7.NS.1.b 7.NS.1.c 7.NS.1.d	1	This lesson demonstrates different models that represent subtraction of integers using a real-world situation, two-color counters, and number lines. There is a continued emphasis to understand the power of zero and absolute value. Questions ask students to notice patterns when subtracting integers, as well as the relationship between integer addition and subtraction.	X	X	X	X	
4.5	Adding and Subtracting Rational Numbers	7.NS.1.b 7.NS.1.c 7.NS.1.d	1	This lesson extends the understanding of addition and subtraction of integers over the set of rational numbers. Questions ask students to restate the rules for adding and subtracting signed numbers, and then demonstrate their understanding.			X		

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Skills Practice Correlation for Chapter 4

Lessons		Problem Set	Objective(s)
4.1	Using Models to Understand Integers	1 - 10	Determine ending positions by adding and subtracting integers
		11 - 20	Write number sentences
		21 - 30	Calculate sums
		31 - 38	Write number sentences to describe the roll of two number cubes
		39 - 46	Create and write number sentences that result in 10
4.2	Adding Integers, Part I	1 - 8	Use a number line to determine unknown numbers
		9 - 14	Use number lines to determine sums
		15 - 20	Write absolute values for integer pairs
		21 - 28	Complete number line models to determine unknown addends
4.3	Adding Integers, Part II		Vocabulary
		1 - 6	Determine sums using number lines and two-color counters
		7 - 14	Write number sentences to represent two-color counter models
		15 - 22	Draw two-color counters to represent and solve number sentences
		23 - 30	Complete two-color counter models to determine unknown addends
		31 - 40	Determine sums
		41 - 48	Determine unknown addends
4.4	Subtracting Integers		Vocabulary
		1 - 6	Draw models to represent integers
		7 - 14	Complete two-color counter models to determine differences
		15 - 22	Use number lines to determine differences
		23 - 30	Rewrite subtraction expressions as addition and solve
		31 - 40	Determine unknown integers
		41 - 48	Determine absolute values
4.5	Adding and Subtracting Rational Numbers	1 - 10	Calculate sums
		11 - 20	Calculate differences
		21 - 30	Add or subtract using algorithms

4.1 Math Football

Using Models to Understand Integers

Learning Goals

In this lesson, you will:

- Represent numbers as positive and negative integers.
- Use a model to represent the sum of a positive and a negative integer.

Essential Ideas

- A model can be used to represent the sum of a positive and negative integer.
- Information from a model can be rewritten as a number sentence.

Common Core State Standards for Mathematics

7.NS The Number System

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
   a. Describe situations in which opposite quantities combine to make 0.
   b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

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Overview

A math football game is used to model the sum of a positive and negative integer. Rules for the game and a game board are provided. Students use number cubes to generate the integers. They will then take that same information and write integer number sentences. If needed, nets for two cubes are provided on the last page of this lesson.

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Warm Up

Compute the sum of two temperatures. A thermometer can be used to determine each temperature sum if needed.

1. $-2° + 13°$

   11°

2. $-5° + 32°$

   27°

3. $-42° + 17°$

   $-25°$

4. $-14° + 6°$

   $-8°$

5. $-2° + 2°$

   0°

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4.1 MATH FOOTBALL

Using Models to Understand Integers

Learning Goals

In this lesson, you will:

- Represent numbers as positive and negative integers.
- Use a model to represent the sum of a positive and a negative integer.

Golfers like negative numbers. This is because, in golf, the lower the score, the better the golfer is playing. Runners like negative numbers too. They often split the distances they have to run into two or more equal distances. If they are on pace to win, they will achieve what is called a negative split.

What about football? What are some ways in which negative numbers can be used in that sport?

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Problem 1

With a partner, students play math football. Two number cubes are used to generate movement on the game board, and if needed, nets for two cubes are provided on the last page of this lesson. One cube generates the number of yard lines moving up the field and the second cube generates yard lines moving down the field. After playing a game, students will answer questions based on their game experience.

Grouping

Ask a student to read the introduction before Question 1 aloud. Discuss the rules and scoring procedures and complete Question 1 as a class.

Discuss Phase, Introduction

- Where does each player start?
- Which cube tells you how many yards to move up the field?
- Which cube tells you how many yards to move down the field?
- When is halftime?
- Do you switch goals at halftime?
- When is the game over?
- How does a player score 6 points?
- How does a player lose 2 points?

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Problem 1 Hut! Hut! Hike!

You and a partner are going to play "Math Football." You will take turns rolling two number cubes to determine how many yards you can advance the football toward your end zone.

Player 1 will be the Home Team and Player 2 will be the Visiting Team. In the first half, the Home Team will move toward the Home end zone, and the Visiting Team will move toward the Visiting end zone.

Rules:

Players both start at the zero yard line and take turns. On your turn, roll two number cubes, one red and one black. The number on each cube represents a number of yards. Move your football to the left the number of yards shown on the red cube. Move your football to the right the number of yards shown on the black cube. Start each of your next turns from the ending position of your previous turn.

(Nets are provided at the end of the lesson so you can cut out and construct your own number cubes. Don't forget to color the number cubes black and red.)

Scoring:

Each player must move the football the combined value of both number cubes to complete each turn and be eligible for points. When players reach their end zone, they score 6 points. If players reach their opponent's end zone, they lose 2 points. An end zone begins on either the +10 or −10 yard line.

Example:

	Player	Starting Position	Results of the Number Cubes Roll	Ending Position
First Turn	Home Team	0	Red 3 and Black 5	+2
	Visiting Team	0	Red 5 and Black 6	+1
Second Turn	Home Team	+2	Red 1 and Black 6	+7
	Visiting Team	+1	Red 6 and Black 2	−3

1. Read through the table. After two turns, which player is closest to their end zone?

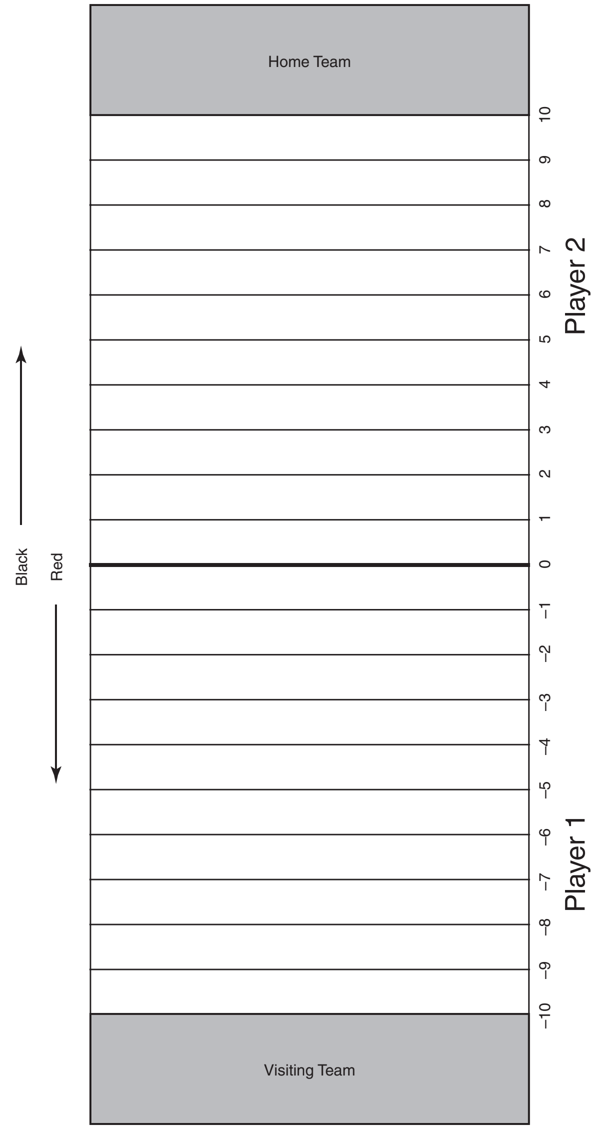
   The Home Team player is closest to the Home end zone.

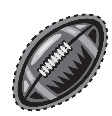
- Where are the end zones?
- What happens if the numbers you roll take you further than the end zone? Do you still score 6 points?

Grouping

Have students play Math Football with a partner.

2. Let's play Math Football. Begin by selecting the home or visiting team. Then, cut out your football. Set a time limit for playing a half. You will play two halves. Make sure to switch ends at half-time with the Home Team moving toward the Visiting end zone, and the Visiting Team moving toward the Home end zone.





Note

This page is intentionally left blank so students can remove the Math Football game board and cut out the footballs.

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Grouping

Have students complete Question 3 with a partner. Then share the responses as a class.

Share Phase, Question 3

- Why do you want the black cube to show the greater value when approaching the Home Team end zone?
- Why do you want the red cube to show the greater value when approaching the Away Team end zone?
- What roll would cause you to move backwards?
- What roll would cause you to move forwards?
- What roll would cause you to have no gain in yardage?
- What is an example of two values that would send you back to the yard line where you began?
- What roll would cause you to move the least distance?
- What roll would cause you to move the greatest distance?

3. Answer each question based on your experiences playing Math Football.

a. When you were trying to get to the Home end zone, which number cube did you want to show the greater value? Explain your reasoning.

As I moved toward the Home end zone, I wanted the black cube to show the greater value. When the value on the black cube was greater, my football moved to the right.

b. When you were trying to get to the Visiting end zone, which number cube did you want to show the greater value? Explain your reasoning.

As I moved toward the Visiting end zone, I wanted the red cube to show the greater value. When the value on the red cube was greater, my football moved to the left.

c. Did you ever find yourself back at the same position you ended on your previous turn? Describe the values shown on the cubes that would cause this to happen.

If I rolled the same number on both number cubes, I could not move to the right or left for that turn. For example, if I rolled a 2 on both the red and black number cubes, I moved to the right 2, then I moved to the left 2, and ended up where I started.

d. Describe the roll that could cause you to move your football the greatest distance either left or right.

When I roll a 6 on one number cube and a 1 on the other number cube, my football could move five spaces.

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Problem 2

Moves on the football field from the previous problem are changed into number sentences. Each number sentence contains both positive and negative integers and students will combine positive and negative integers to answer related questions.

Grouping

Ask a student to read the introduction before Question 1 aloud. Discuss this information and complete Question 1 as a class.

Discuss Phase, Table

- Which team player had better field position after the first turn?
- How do you decide which team player has better field position?
- Which team player had better field position after the second turn?

Discuss Phase, Question 1

- What is a number sentence that represents the first turn of the Home Team player?
- What is a number sentence that represents the first turn of the Visiting Team player?
- What is a number sentence that represents the second turn of the Home Team player?

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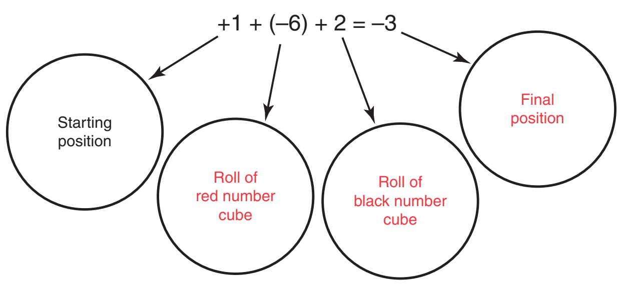
Problem 2 Writing Number Sentences

You can write number sentences to describe the results of number cube rolls. Think of the result of rolling the red number cube as a negative number and the result of rolling the black number cube as a positive number.

Consider the example from Problem 1. The number sentence for each turn has been included.

	Player	Starting Position	Results of the Number Cubes Roll	Ending Position	Number Sentence
First Turn	Home Team	0	Red 3 and Black 5	+2	$0 + (-3) + 5 = +2$
	Visiting Team	0	Red 5 and Black 6	+1	$0 + (-5) + 6 = +1$
Second Turn	Home Team	+2	Red 1 and Black 6	+7	$+2 + (-1) + 6 = +7$
	Visiting Team	+1	Red 6 and Black 2	−3	$+1 + (-6) + 2 = -3$

1. Describe each part of the number sentence for the second turn of the Visiting Team player.



Grouping

Have students complete Question 2 with a partner. Then share the responses as a class.

Share Phase, Question 2

- What end zone is the Home Team player closest to?
- What end zone is the Visiting Team player closest to?
- What end zone do you want to be closest to?
- If an integer is added to its opposite, what is the result?
- Are the results always the same for all integers when they are added to their opposite?

2. Write a number sentence for each situation. Use the game board for help.

a. The Home Team player starts at the zero yard line and rolls a red 6 and a black 2. What is the ending position?

Number sentence $0 + (-6) + 2 = -4$

b. The Visiting Team player starts at the zero yard line and rolls a red 5 and a black 4. What is the ending position?

Number sentence $0 + (-5) + 4 = -1$

c. The Home Team player starts at the 5 yard line and rolls a red 2 and a black 2. What is the ending position?

Number sentence $5 + (-2) + 2 = 5$

d. The Visiting Team player starts at the -5 yard line and rolls a red 4 and a black 6. What is the ending position?

Number sentence $-5 + (-4) + 6 = -3$

e. Suppose the Home Team player is at the $+8$ yard line. Complete the table and write two number sentences that will put the player into the Home end zone.

Starting Position	Roll of the Red Number Cube	Roll of the Black Number Cube	Number Sentence
$+8$	-1	$+3$	$+8 + (-1) + 3 = 10$
$+8$	-2	$+5$	$+8 + (-2) + 5 = 11$

f. Suppose the Visiting Team player is at the -8 yard line. Complete the table and write two number sentences that will put the player into the Visiting end zone.

Starting Position	Roll of the Red Number Cube	Roll of the Black Number Cube	Number Sentence
-8	-4	$+2$	$-8 + (-4) + 2 = -10$
-8	-6	$+3$	$-8 + (-6) + 3 = -11$

Be prepared to share your solutions and methods.

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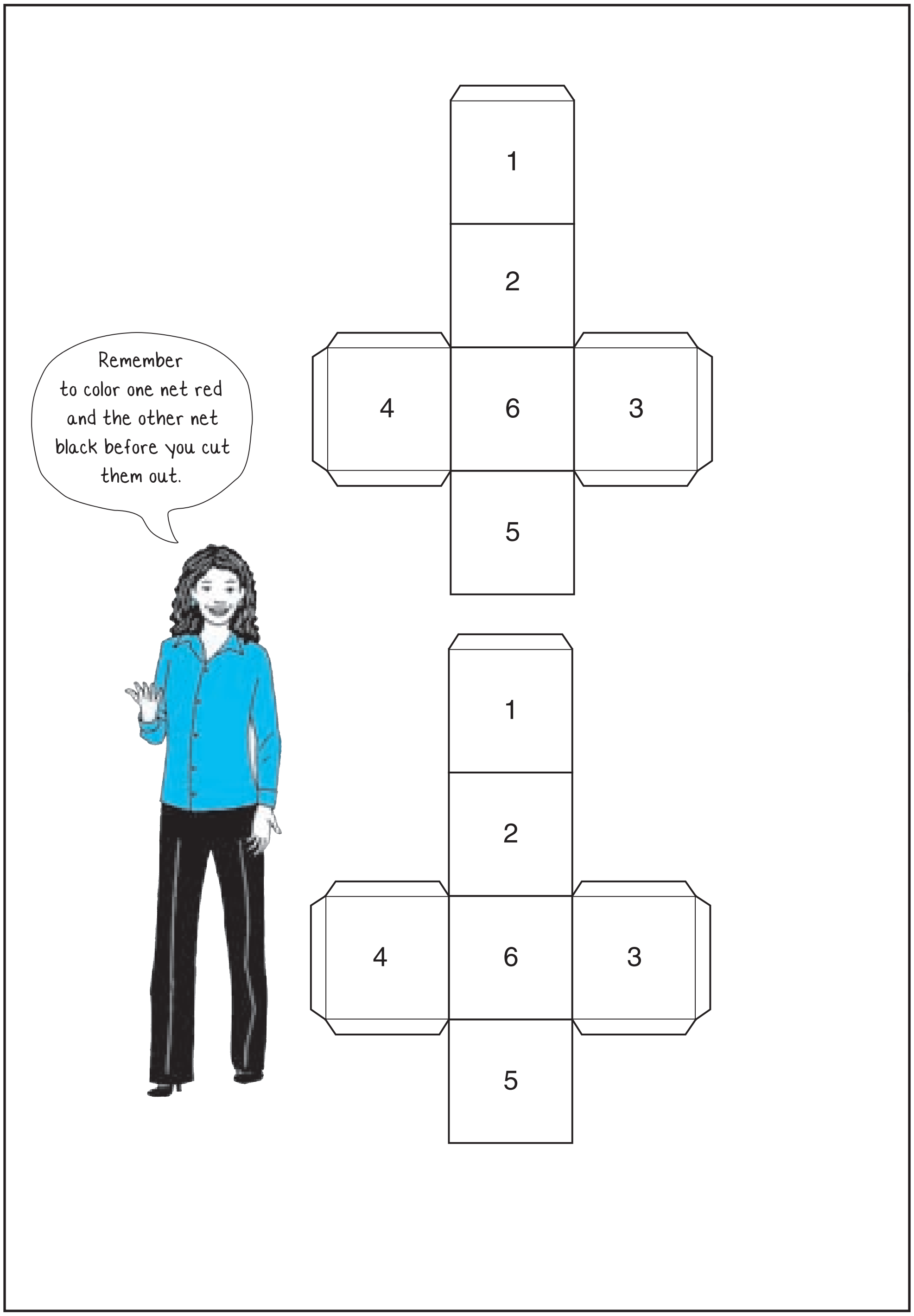
Note

Number cubes are provided for Math Football. Remind students to color one net red and the other net black before cutting them out.



Follow Up

Assignment

Use the Assignment for Lesson 4.1 in the Student Assignments book. See the Teacher's Resources and Assessments book for answers.

Skills Practice

Refer to the Skills Practice worksheet for Lesson 4.1 in the Student Assignments book for additional resources. See the Teacher's Resources and Assessments book for answers.

Assessment

See the Assessments provided in the Teacher's Resources and Assessments book for Chapter 4.

Check for Students' Understanding

Welcome to the You Do The Math Hotel! This large hotel has a ground floor (street level), 26 floors of guest rooms above street level, and 5 floors of parking below street level. The hotel's elevator is able to stop on every floor. In this hotel, street level is represented by zero on a number line.

1. Draw a diagram of the hotel's elevator.

The diagram of the hotel should reflect 26 floors above street level and the 5 floors of parking below street level.

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2. Thinking of the height of the building as a number line, describe the street level.

The street level should be equivalent to zero on a number line.

3. Suppose the elevator starts at street level, goes up 7 floors, and then goes down 12 floors. What floor is the elevator on?

The elevator would be on the 5th floor of the parking garage.

4. Suppose the elevator starts at street level, goes up 10 floors, and then goes down 3 floors. What floor is the elevator on?

The elevator would be on the 7th floor.

5. Suppose the elevator starts at street level, goes down 4 floors, and then goes up 11 floors. What floor is the elevator on?

The elevator would be on the 7th floor.

6. Suppose the elevator starts at street level, goes down 2 floors, and then goes up 5 floors, and finally goes down 3 floors. What floor is the elevator on?

The elevator would be on street level.

4.2 Walk the Line

Adding Integers, Part 1

Learning Goals

In this lesson, you will:

- Model the addition of integers on a number line.
- Develop a rule for adding integers.

Essential Ideas

- On a number line, when adding a positive integer, move to the right.
- One a number line, when adding a negative integer, move to the left.
- When adding two positive integers, the sign of the sum is always positive.
- When adding two negative integers, the sign of the sum is always negative.
- When adding a positive and a negative integer, the sign of the sum is the sign of the number that is the greatest distance from zero on the number line.

Common Core State Standards for Mathematics

7.NS The Number System

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
   b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

Overview

A number line is used to model the sum of two integers. Through a series of activities, students will notice patterns for adding integers. The first activity is a game played in partners called "What's My Number?" In the second activity students use a number line to determine the number described by a word statement.

Students will determine the sum of two integers moving left and right on a number line. Questions focus students on the distance an integer is from zero on the number line, or the absolute value of the integer, to anticipate writing a rule for the sum of two integers having different signs. Students will then write the rules for the sum of any two integers.

Note

You may want to consider taping a number line to the floor of your classroom so students can "walk the line" to physically act out the models they will create throughout this chapter. In this lesson, all the number sentences are addition; in lesson 4.4 the number sentences involve subtraction.

To "walk the number line," for either an addition or subtraction sentence, have the student start at zero and walk to the value of the first term of the expression. When adding, the student should turn to the right and walk forward if adding a positive number, or walk backward if adding a negative number. When subtracting, the student should turn to the left and walk forward if adding a positive number, or walk backward if adding a negative number.

Warm Up

Remember the You Do The Math Hotel? This large hotel has a ground floor (street level) and 26 floors of guest rooms above street level and 5 floors of parking below street level. The hotel's elevator can stop at every floor. In this hotel, street level is represented by zero on a number line.

You can assign positive integers to the floors above street level and negative integers to the floors below street level. Write an integer addition problem that models the elevator's motion in each a case.

1. The elevator starts at street level, goes up 7 floors, and then goes down 3 floors.
   $0 + 7 + (-3)$

2. The elevator starts at street level, goes up 10 floors, and then goes down 12 floors.
   $0 + 10 + (-12)$

3. The elevator starts at street level, goes down 4 floors, and then goes up 11 floors.
   $0 + (-4) + 11$

4. The elevator starts at street level, goes down 2 floors, and then goes up 5 floors and finally goes down 3 floors.
   $0 + (-2) + 5 + (-3)$

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4.2 Walk the Line

Adding Integers, Part I

Learning Goals

In this lesson, you will:

- Model the addition of integers on a number line.
- Develop a rule for adding integers.

Corinne: "I'm thinking of a number between −20 and 20. What's my number?"

Benjamin: "Is it −5?"

Corinne: "Lower."

Benjamin: "−2?"

Corinne: "That's not lower than −5."

Benjamin: "Oh, right. How about −11?"

Corinne: "Higher."

Benjamin: "−8?"

Corinne: "Lower."

Benjamin: "−9?"

Corinne: "You got it!"

Try this game with a partner. See who can get the number with the fewest guesses.

Problem 1

Several word statements are given and students use a number line to determine the integer described by each statement and explain their reasoning. Two examples of adding integers on a number line are provided and students answer questions that describe the steps taken to compute the sum of the integers. They will use number lines to compute the sum of both positive and negative integers. Questions focus on the distance the integer is from zero (absolute value). Finally, students write rules for the addition of integers through a series of questions.

Grouping

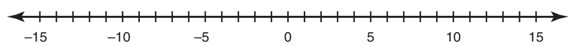
Have students complete Question 1 with a partner. Then share the responses as a class.

Share Phase, Question 1

- How do you write '7 more than −9' using math symbols?
- Do you move left or right on the number line to compute '7 more than −9'?
- How do you write '2 more than −6' using math symbols?
- Do you move left or right on the number line to compute '2 more than −6'?
- How do you write '10 more than 6' using math symbols?
- Do you move left or right on the number line to compute '10 more than 6'?

Problem 1 Adding on Number Lines

1. Use the number line and determine the number described by each. Explain your reasoning.



a. the number that is 7 more than −9

The number that is 7 more than −9 is −2. Go to −9 on the number line, and then move 7 units to the right.

b. the number that is 2 more than −6

The number that is 2 more than −6 is −4. Go to −6 on the number line, and then move 2 units to the right.

c. the number that is 10 more than −8

The number that is 10 more than −8 is 2. Go to −8 on the number line, and then move 10 units to the right.

d. the number that is 10 less than 6

The number that is 10 less than 6 is −4. Go to 6 on the number line, and then move 10 units to the left.

e. the number that is 5 less than −4

The number that is 5 less than −4 is −9. Go to −4 on the number line, and then move 5 units to the left.

f. the number that is 2 less than −4

The number that is 2 less than −4 is −6. Go to −4 on the number line, and then move 2 units to the left.

- How do you write '5 less than −4' using math symbols?
- Do you move left or right on the number line to compute '5 less than −4'?
- How do you write '2 less than −4' using math symbols?
- Do you move left or right on the number line to compute '2 less than −4'?
- What do the words 'more than' imply in a word statement with respect to a number line?
- What do the words 'less than' imply in a word statement with respect to a number line?

Grouping

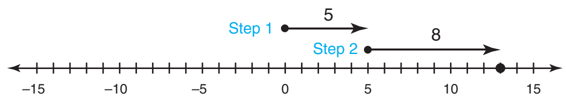
- Ask a student to read the information in the worked example aloud. Discuss the information as a class.
- Have students complete Questions 2 and 3 with a partner. Then share the responses as a class.

Share Phase, Question 2

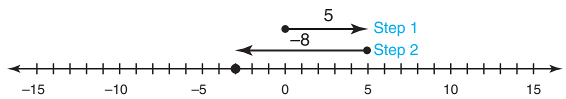
- When computing the sum of two or move integers using a number line, where do you always start?
- When computing the sum of two or move integers using a number line, when you start at zero, how do you know which direction, left or right, to move next?
- How do you know which direction, left or right, to move, to combine the second term?
- On a number line, what is the sign of the first term, if you move from zero on the number line, to the left?
- On a number line, what is the sign of the first term, if you move from zero on the number line, to the right?

A number line can be used to model integer addition.
When adding a positive integer, move to the right on a number line.
When adding a negative integer, move to the left on a number line.

Example 1: The number line shows how to determine 5 + 8.



Example 2: The number line shows how to determine 5 + (−8).



2. Compare the first steps in each example.

a. What distance is shown by the first term in each example?

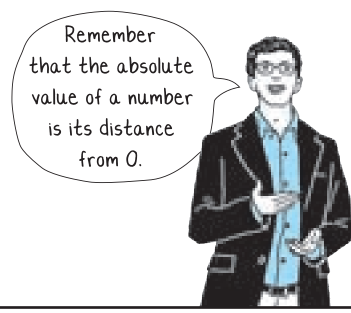
The distance shown by the first term in each example is the same: 5 units.

b. Describe the graphical representation of the first term. Where does it start and in which direction does it move? Why?

The graphical representation for the first term begins at 0 and moves to the right. It moves to the right because the first term is positive.

c. What is the absolute value of the first term in each example?

The absolute value of 5 is 5.



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Share Phase, Question 3

- On a number line, what is the sign of the second term, if you move from the location of the first term, to the left?
- On a number line, what is the sign of the second term, if you move from the location of the first term, to the right?
- What information does the absolute value of a term give you, with respect to the graphical representation?

Grouping

Have students complete Questions 4 through 6 with a partner. Then share the responses as a class.

Share Phase, Question 4

- If the sign of the first term was positive and the sign of the second term was positive, which directions did you move on the number line?
- If the sign of the first term was positive and the sign of the second term was negative, which directions did you move on the number line?
- If the sign of the first term was negative and the sign of the second term was negative, which directions did you move on the number line?

3. Compare the second steps in each example.

   a. What distance is shown by the second term in each example?

   The distance shown by the second term in each example is the same: 8 units.

   b. Why did the graphical representation for the second terms both start at the endpoints of the first terms but then continue in opposite directions? Explain your reasoning.

   The arrows are drawn in opposite directions because the numbers are opposites of each other. Positive 8 tells me to move to the right; negative 8 tells me to go in the opposite direction, or move to the left.

   c. What are the absolute values of the second terms?

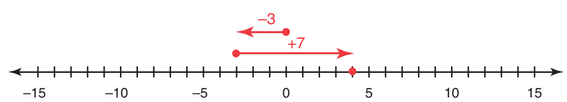
   $|8| = 8$

   $|-8| = 8$

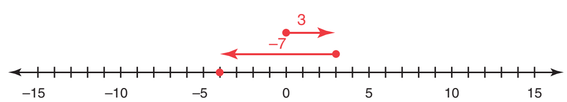
   The absolute values are both 8.

4. Use the number line to determine each sum. Show your work.

   a. $-3 + 7 =$ 4



   b. $3 + (-7) =$ −4

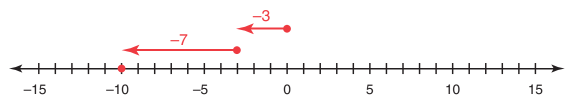


- If the sign of the first term was positive and the sign of the second term was positive, which directions did you move on the number line?

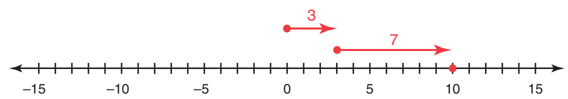
Share Phase, Questions 5 and 6

- Why does moving to the left on a number line leave you with a smaller number?
- Why does moving to the right on a number line leave you with a larger number?
- What information does the absolute value of each term give you?
- The absolute value of the two integers used in each part of this question are the same, why aren't the sums of the two integers the same?

c. $-3 + (-7) =$ <u>-10</u>



d. $3 + 7 =$ <u>10</u>



5. Notice that the first term in each expression in parts (a) through (d) was either 3 or (-3).

a. What do you notice about the distances shown by these terms on the number lines?

The distances are the same: 3 units.

b. What is the absolute value of each term?

$|3| = 3$

$|-3| = 3$

The absolute values are equal: 3.

6. Notice that the second term in each expression was either 7 or (-7).

a. What do you notice about the distances shown by these terms on the number lines?

The distances are the same: 7 units.

b. What is the absolute value of each term?

$|7| = 7$

$|-7| = 7$

The absolute values are equal: 7.

4

Grouping

Have students complete Questions 7 through 9 with a partner. Then share the responses as a class.

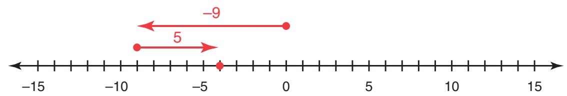
Share Phase, Questions 7 through 9

- Can you think of a rule you might use when working with a number line to add two positive integers?
- Can you think of a rule you might use when working with a number line to add two negative integers?
- Can you think of a rule you might use when working with a number line to add a positive and a negative integer?
- Could a number line be used to compute the sum of more than two integers?
- Can you think of a way to solve these problems without using a number line?

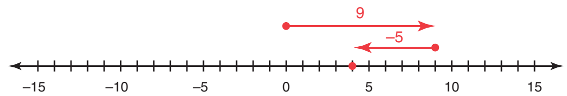
4

7. Use the number line to determine each sum. Show your work.

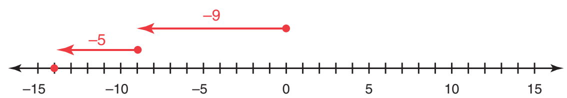
a. $-9 + 5 =$ <u>-4</u>



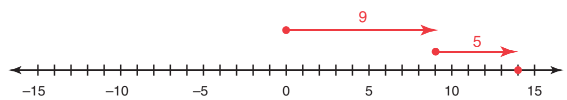
b. $9 + (-5) =$ <u>4</u>



c. $-9 + (-5) =$ <u>-14</u>



d. $9 + 5 =$ <u>14</u>



8. Notice that the first term in each expression in parts (a) through (d) was either 9 or (−9).

   a. What do you notice about the distances shown by these terms on the number lines?

   The distances are the same: 9 units.

   b. What is the absolute value of each term?

   $|9| = 9$

   $|-9| = 9$

   The absolute values are equal: 9.

9. Notice that the second term in each expression was either 5 or (−5).

   a. What do you notice about the distances shown by these terms on the number lines?

   The distances are the same: 5 units.

   b. What is the absolute value of each term?

   $|5| = 5$

   $|-5| = 5$

   The absolute values are equal: 5.

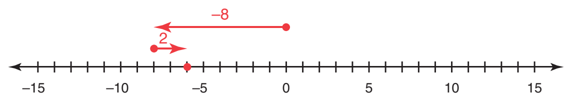
4

Grouping

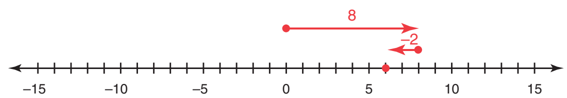
Have students complete Questions 10 through 13 with a partner. Then share the responses as a class.

10. Use the number line to determine each sum. Show your work.

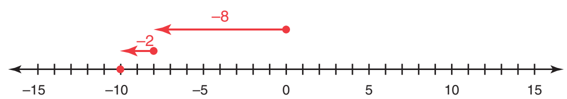
a. $-8 + 2 =$ <u>-6</u>



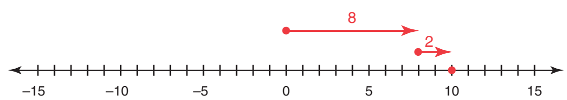
b. $8 + (-2) =$ <u>6</u>



c. $-8 + (-2) =$ <u>-10</u>

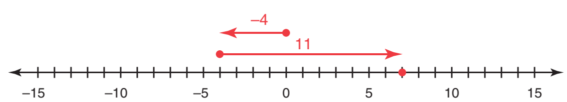


d. $8 + 2 =$ <u>10</u>



11. Use the number line to determine each sum. Show your work.

a. $-4 + 11 =$ <u>7</u>



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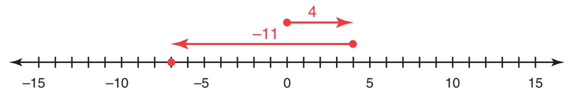
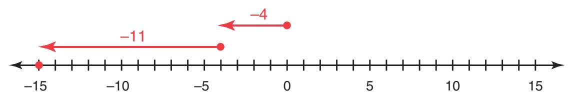
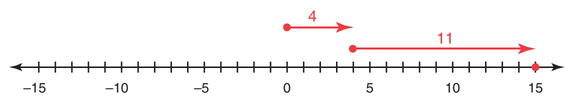
b. $4 + (-11) =$ <u>−7</u>



c. $-4 + (-11) =$ <u>−15</u>



d. $4 + 11 =$ <u>15</u>



Share Phase, Question 12

- When is the sum of a positive number and a negative number a positive answer?
- When is the sum of a positive number and a negative number a negative answer?
- When is the sum of a positive number and a positive number a positive answer?
- When is the sum of a positive number and a positive number a negative answer?
- When is the sum of a negative number and a negative number a positive answer?
- When is the sum of a negative number and a negative number a negative answer?

12. In Questions 4 through 11, what patterns do you notice when:

a. you are adding two positive numbers?

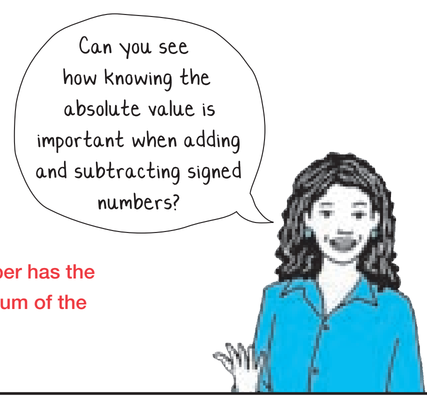
The sum is always positive.

b. you are adding two negative numbers?

The sum is always negative.

c. you are adding a negative and a positive number?

When the negative number has the greatest distance from zero, the sum of the two numbers is negative. When the positive number has the greatest distance from zero, the sum of the two numbers is positive.



4

Grouping

Have students complete Question 13 with a partner. Then share the responses as a class.

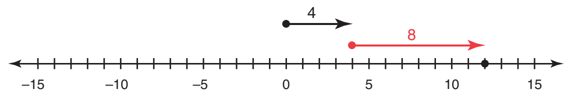
Share Phase, Question 13

- When combining two integers, if you are always moving to the left, what does this tell you about the sign of the answer?
- When combining two integers, if you are always moving to the right, what does this tell you about the sign of the answer?
- When combining two integers, if you are moving to the left and then moving to the right, what does this tell you about the sign of the answer?
- When combining two integers, if you are moving to the left and then moving more to the right, what does this tell you about the sign of the answer?
- When combining two integers, if you are moving to the right and then moving more to the left, what does this tell you about the sign of the answer?

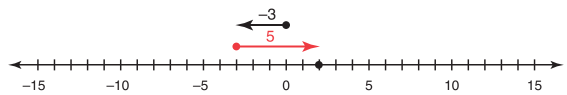
4

13. Complete each number line model and number sentence.

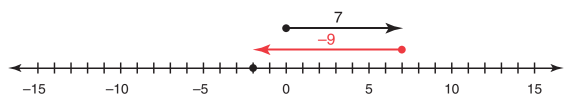
a. $4 + \underline{\quad 8 \quad} = 12$



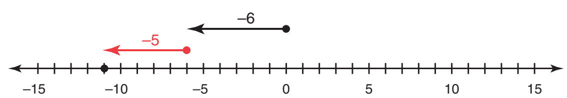
b. $-3 + \underline{\quad 5 \quad} = 2$



c. $7 + \underline{\quad -9 \quad} = -2$



d. $-6 + \underline{\quad -5 \quad} = -11$



Be prepared to share your solutions and methods.

Follow Up

Assignment

Use the Assignment for Lesson 4.2 in the Student Assignments book. See the Teacher's Resources and Assessments book for answers.

Skills Practice

Refer to the Skills Practice worksheet for Lesson 4.2 in the Student Assignments book for additional resources. See the Teacher's Resources and Assessments book for answers.

Assessment

See the Assessments provided in the Teacher's Resources and Assessments book for Chapter 4.

Check for Students' Understanding

Welcome back to the You Do The Math Hotel! We hope you enjoy your stay! You may remember, this hotel has a ground floor and 26 floors of guest rooms above street level and 5 floors of parking below street level. The hotel's elevator can stop at every floor.

Write a sentence to describe the motion of the elevator modeled by each integer addition problem below. Then, state the floor on which the elevator started, and compute the sum to determine on which floor the elevator stops.

1. $(-2) + 20$

The elevator started on the 2nd floor of the garage and went up 20 floors, to floor 18.

2. $12 + (-7)$

The elevator started on the 12th floor and went down 7 floors, to floor 5.

3. $2 + (-5)$

The elevator started on the 2nd floor and went down 5 floors, to the 3rd floor of the garage.

4. $(-5) + 15$

The elevator started on the 5th floor of the garage and went up 15 floors, to floor 10.

5. $26 + (-20) + (-5) + (-3)$

The elevator started on the 26th floor and went down 20 floors, then down 5 more floors, then down 3 more floors, to the 2nd floor of the garage.

4

4

4.3 Two-Color Counters

Adding Integers, Part II

Learning Goals

In this lesson, you will:

- Model the addition of integers using two-color counters.
- Develop a rule for adding integers.

Key Term

- additive inverses

Essential Ideas

- Two numbers with the sum of zero are called additive inverses.
- Addition of integers is modeled using two-color counters that represent positive charges (yellow counters) and negative charges (red counters).
- When two integers have the same sign and are added together, the sign of the sum is the sign of both integers.
- When two integers have the opposite sign and are added together, the integers are subtracted and the sign of the sum is the sign of the integer with the greater absolute value.

Common Core State Standards for Mathematics

7.NS The Number System

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
   a. Describe situations in which opposite quantities combine to make 0.
   b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
   c. Understand subtraction of rational number as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

4

Overview

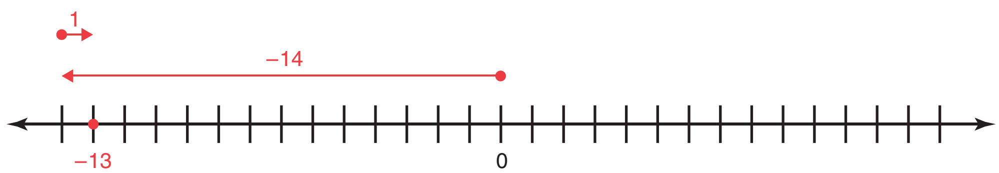
Two-color counters are used to model the sum of two integers. Through a series of activities, students will develop rules for adding integers. Students determine the sum of two integers with opposite signs using the number line model. The term additive inverse is defined. Examples of modeling the sum of two integers with opposite signs using two-color counters is provided and the counters are paired together, one positive counter with one negative counter, until no possible pairs remain. The resulting counters determine the sum of the integers.

In the second activity, several models are given and students write a number sentence to represent each model. Students draw a model for each given number sentence to determine the sum of the two integers. Questions focus students to write rules for how to determine the sum of any two integers. They will create a graphic organizer to represent the sum of additive inverses using number sentences in words, a number line model, and a two-color counter model. Students then describe the general representation of zero in each model.

Warm Up

Use a number line to determine each sum. Then write a sentence to describe the movement you used on the number line to compute the sum of the two integers.

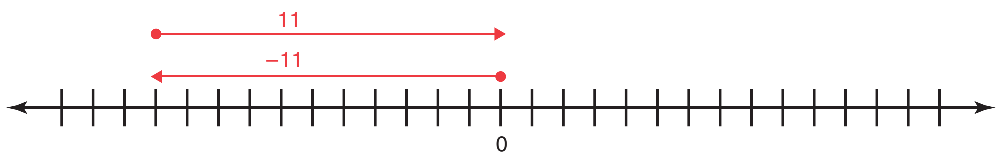
1. $(-14) + 1$



−13

I started at zero, then moved left to 14, then moved right 1, to stop at −13.

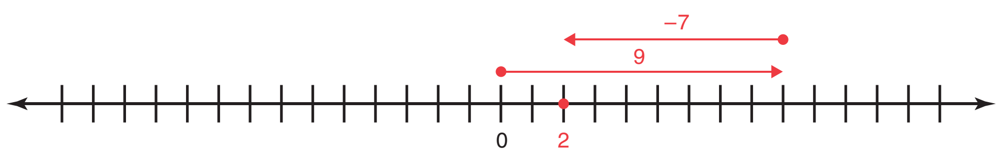
2. $(-11) + 11$



0

I started at zero, then moved left to 11, then moved right 11, to stop at 0.

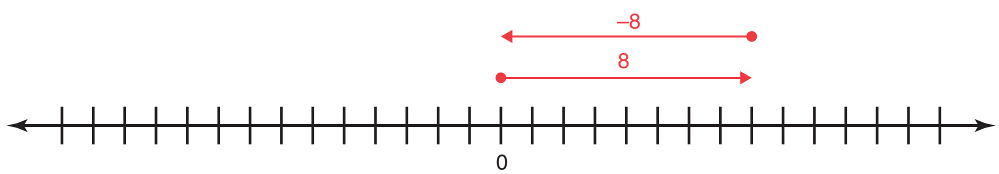
3. $9 + (-7)$



2

I started at zero, then moved right to 9, then moved left 7, to stop at 2.

4. $8 + (-8)$



0

I started at zero, then moved right to 8, then moved left 8, to stop at 0.

4

4

4.3 Two-Color Counters

Adding Integers, Part II

Learning Goals

In this lesson, you will:

- Model the addition of integers using two-color counters.
- Develop a rule for adding integers.

Key Term

- additive inverses

Opposites are all around us. If you move forward two spaces in a board game and then move back in the opposite direction two spaces, you're back where you started. In tug-of-war, if one team pulling on the rope pulls exactly as hard as the team on the opposite side, no one moves. If an element has the same number of positively charged protons as it does of negatively charged electrons, then the element has no charge.

In what ways have you worked with opposites in mathematics?

4

Problem 1

Students determine the sum of two integers using a number line model. The additive inverse is defined as two numbers with the sum of zero. Two-color counters that represent positive charges (+) and negative charges (−) are used to model the sum of two integers. Examples using this model are provided and students will create an alternate model to represent the same sum. They are given two-color counter models and will write a number sentence to describe each model. Students then create two-color counter models for each of several given number sentences. Questions focus students to write rules to determine the sum of any two integers that have the same sign, and the sum of any two integers that have opposite signs. The rules are used to determine the sum in each of several number sentences.

Grouping

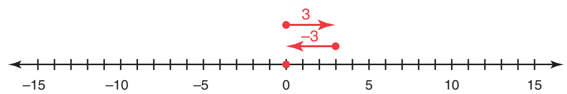
- Have students complete Question 1 with a partner. Then share the responses as a class.
- Ask a student to read the information following Question 1 aloud. Discuss the worked examples and complete Questions 2 and 3 as a class.

4

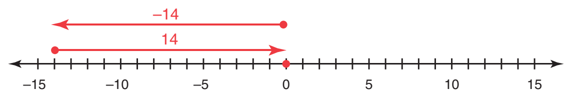
Problem 1 Two-Color Counters

1. Use the number line model to determine each sum.

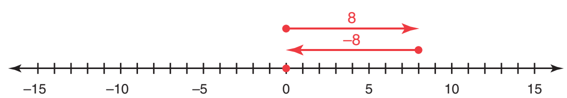
   a. $3 + (-3) =$ 0



   b. $(-14) + 14 =$ 0



   c. $8 + (-8) =$ 0



   d. What pattern do you notice?

   In each part, the numbers are opposites and their sum is 0.

Two numbers with the sum of zero are called **additive inverses**.

Addition of integers can also be modeled using two-color counters that represent positive (+) charges and negative (−) charges. One color, usually red, represents the negative number, or negative charge. The other color, usually yellow, represents the positive number, or positive charge. In this book, gray shading will represent the negative number, and no shading will represent the positive number.

(−) = −1   (+) = +1

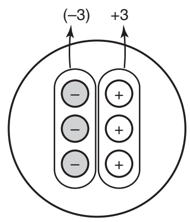
Share Phase, Question 1

- What is the sum of any integer and its opposite?
- Why is the sum of any integer and its opposite always equal to zero?
- What is an example in real life of combining something with its opposite?

Discuss Phase, Questions 2 and 3

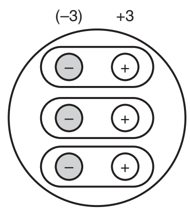
- When computing the sum of two integers using a two-color counter model, if the sum is zero, what is true about the number of positive charges (+) and the number of negative charges (−)?
- When computing the sum of two integers using a two-color counter model, what is the first step?

You can model the expression 3 + (−3) in different ways using two-color counters:



Three positive charges and three negative charges have no charge.

$3 + (-3) = 0$



Each positive charge is paired with a negative charge.

$3 + (-3) = 0$

2. What is the value of each − and + pair shown in the second model?

The value of each positive and negative pair is 0.

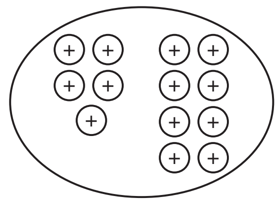
3. Describe how you can change the numbers of − and + counters in the model, but leave the sum unchanged.

I could add 2 more − and 2 more + and the sum would still be zero.

4

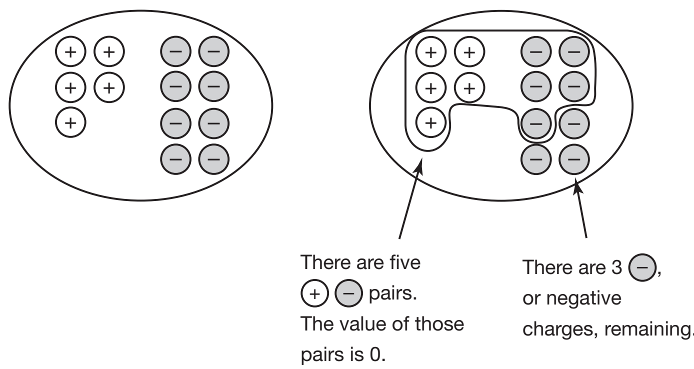
Let's consider two examples where integers are added using two-color counters.

Example 1: 5 + 8



There are 13 positive charges in the model. The sum is 13.

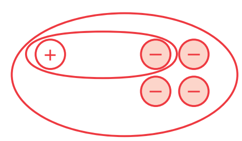
Example 2: 5 + (−8)



There are 3 negative charges remaining. The sum of 5 + (−8) is −3.

4. Create another model to represent a sum of −3. Write the appropriate number sentence.

Answers will vary. 1+ (−4) = −3



Grouping

Have students complete Questions 4 and 5 with a partner. Then share the responses as a class.

Share Phase, Questions 4 and 5

- What is another model to represent a sum of −3?
- How many different models representing a sum of −3 are possible?
- What do all of the models representing a sum of −3 have in common?
- How are all of the models representing a sum of −3 different from each other?

Grouping

Have students complete Questions 6 through 8 with a partner. Then share the responses as a class.

Share Phase, Questions 6 through 8

- Glancing quickly at a two-color counter model, how can you conclude the sum of the two integers will be negative?
- Glancing quickly at a two-color counter model, how can you conclude the sum of the two integers will be positive?
- What do all two-color counter models resulting in a negative sum have in common?
- What do all two-color counter models resulting in a positive sum have in common?
- Given a sum, how many two-color counter models can be created to represent the sum?

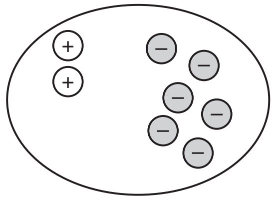
5. Share your model with your classmates. How are they the same? How are they different?

They are the same because each model represents a sum of −3, and each model had 3 more negative counters in it than positive counters.

They are different because everyone chose different numbers to represent the positive and negative counters.

6. Write a number sentence to represent each model.

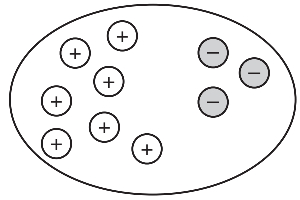
a.



$2 + (-6) = -4$

$-6 + 2 = -4$

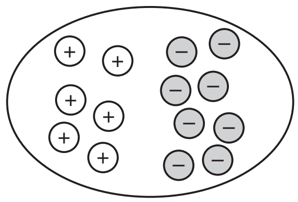
b.



$-3 + 7 = 4$

$7 + (-3) = 4$

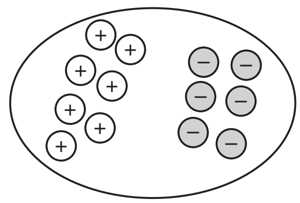
c.



$-8 + 6 = -2$

$6 + (-8) = -2$

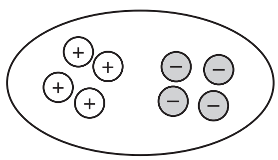
d.



$7 + (-6) = 1$

$-6 + 7 = 1$

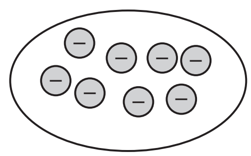
e.



$-4 + 4 = 0$

$4 + (-4) = 0$

f.



$-8 + 0 = -8$

$0 + (-8) = -8$

Grouping

Have students complete Questions 9 and 10 with a partner. Then share the responses as a class.

Share Phase, Questions 9 and 10

- Is there more than one number sentence that would represent this two-color counter model?
- How many pairs can you circle in this two-color counter model?
- If nothing can be paired in the two-color counter model, what does this mean?
- If nothing can be paired in the two-color counter model, what can you conclude about the sum of the integers?
- Can you determine the sign of the sum without circling the pairs in this two-color counter model?
- Can the two-color counter model be used to add more than two integers? If so, how?

4

7. Does the order in which you wrote the integers in your number sentence matter? How do you know?

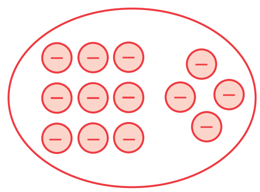
The order doesn't matter because of the Commutative Property of Addition.

8. Write each number sentence in Question 6 a second way.

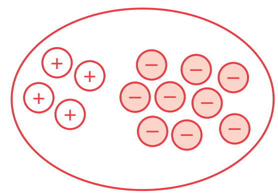
See second number sentence below each model in Question 6.

9. Draw a model for each, and then complete the number sentence.

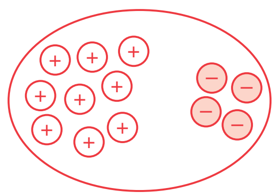
a. $-9 + (-4) =$ $\underline{-13}$



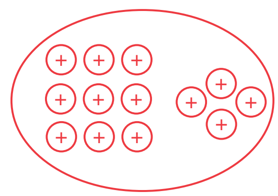
b. $-9 + 4 =$ $\underline{-5}$



c. $9 + (-4) =$ $\underline{5}$

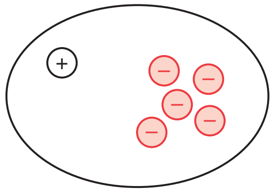


d. $9 + 4 =$ $\underline{13}$

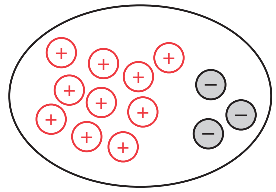


10. Complete the model to determine the unknown integer.

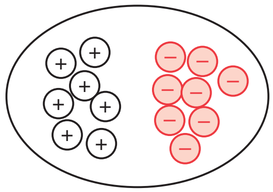
a. $1 + \underline{\ -5\ } = -4$



b. $-3 + \underline{\ 10\ } = 7$



c. $7 + \underline{\ -8\ } = -1$



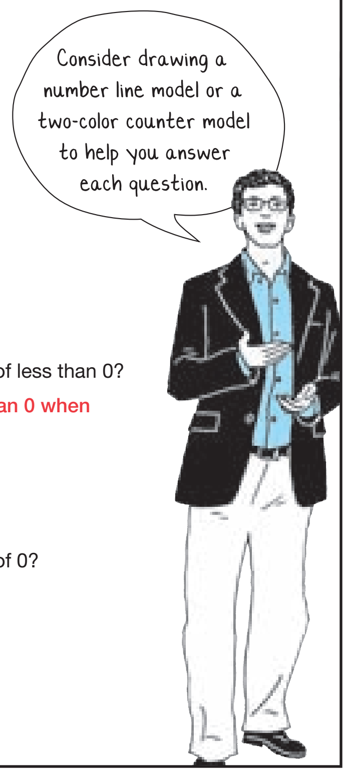
Grouping

Have students complete Questions 11 through 14 with a partner. Then share the responses as a class.

11. Describe the set of integers that makes each sentence true.

a. What integer(s) when added to -7 give a sum greater than 0?

Any integer greater than 7 will give a sum greater than 0 when added to -7.



b. What integer(s) when added to -7 give a sum of less than 0?

Any integer less than 7 will give a sum less than 0 when added to -7.

c. What integer(s) when added to -7 give a sum of 0?

When 7 is added to -7, the sum is 0.

4

12. When adding two integers, what will the sign of the sum be if:

a. both integers are positive?

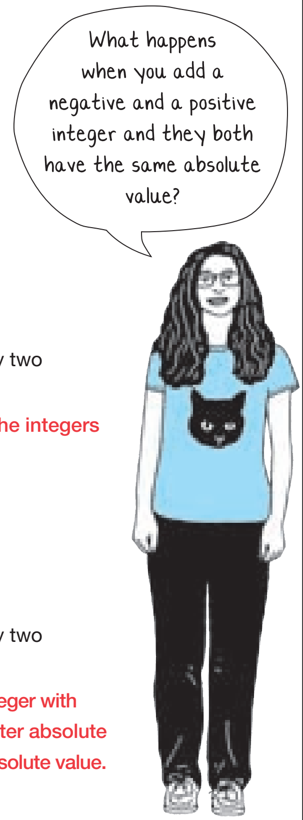
The sign of the sum will be positive.

b. both integers are negative?

The sign of the sum will be negative.

c. one integer is negative and one integer is positive?

The sign of the sum will be the same as the sign of the integer with the greater absolute value, or the sign of the number that is a greater distance away from 0.



Share Phase, Questions 13 and 14

- Is there another way to write this rule? If so, what is it?
- Will this rule work for all integers? Why or why not?

13. Write a rule that states how to determine the sum of any two integers that have the same sign.

When both of the integers have the same sign, I add the integers and keep the sign of the numbers.

14. Write a rule that states how to determine the sum of any two integers that have opposite signs.

When the integers have opposite signs, I subtract the integer with the lesser absolute value from the integer with the greater absolute value and keep the sign of the integer with the greater absolute value.

Grouping

Have students complete Questions 15 and 16 with a partner. Then share the responses as a class.

Share Phase, Questions 15 and 16

- Is it easier to compute the sum or an addend? Why?
- Why wouldn't it be practical to use a two-color counter model to compute this sum?
- Why wouldn't it be practical to use a number line model to compute this sum?
- Glancing at the number sentence, how can you quickly determine the sign of the sum?

Talk the Talk

Students create a graphic organizer to represent the sum of additive inverses by writing a number sentence in words, using a number line to model the integers, and using a two-color counter to model the integers.

Grouping

Have students complete the graphic organizer with a partner. Then share the responses as a class.

15. Use your rule to determine each sum.

a. $-58 + (24) = -34$

b. $-35 + (-15) = -50$

c. $-33 + (-12) = -45$

d. $-48 + 60 = 12$

e. $26 + (-13) = 13$

f. $-67 + 67 = 0$

g. $105 + (-25) = 80$

h. $153 + (-37) = 116$

16. Determine each unknown addend.

a. $\underline{59} + (-25) = 34$

b. $\underline{-14} + 26 = 12$

c. $8 + \underline{-32} = -24$

d. $-12 + \underline{-12} = -24$

e. $-15 + \underline{-13} = -28$

f. $\underline{-21} + 18 = -3$

Talk the Talk

Represent the sum of additive inverses in the graphic organizer provided. First, write a number sentence. Then, represent your number sentence in words, using a number line model, and using a two-color counter model.

Be prepared to share your solutions and methods.

4

NUMBER SENTENCE

$2 + (-2) = 0$

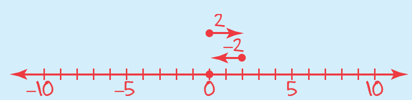
$(-2) + 2 = 0$

IN WORDS

When I add any two opposite numbers, the sum is 0.

ADDITIVE INVERSES AND ZERO, 0

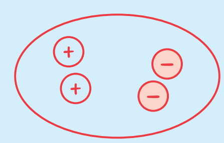
$2 + (-2) = 0$



In any number line model, when the distances of two numbers are equal but in opposite directions, the result is 0.

NUMBER LINE MODEL

$2 + (-2) = 0$



In any two-color counter model, when there are the same number of positive and negative counters, the result is 0.

TWO-COLOR COUNTER MODEL

Follow Up

Assignment

Use the Assignment for Lesson 4.3 in the Student Assignments book. See the Teacher's Resources and Assessments book for answers.

Skills Practice

Refer to the Skills Practice worksheet for Lesson 4.3 in the Student Assignments book for additional resources. See the Teacher's Resources and Assessments book for answers.

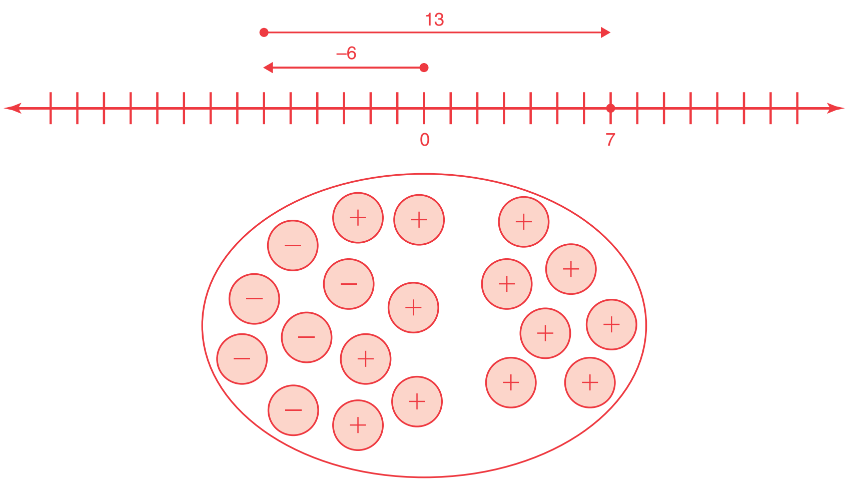
Assessment

See the Assessments provided in the Teacher's Resources and Assessments book for Chapter 4.

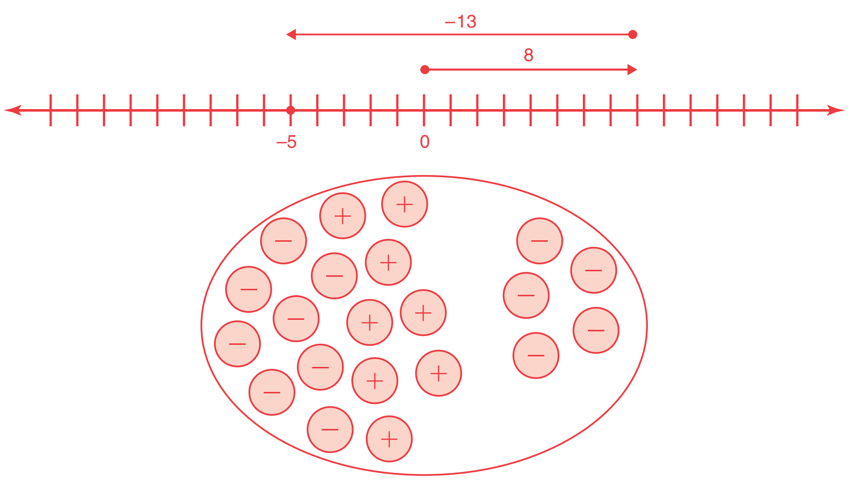
Check for Students' Understanding

Draw both, a model using two-color counters, and a model using a number line, to represent each number sentence. Then, determine the sum.

1. $(-6) + 13 = 7$



2. $8 + (-13) = -5$



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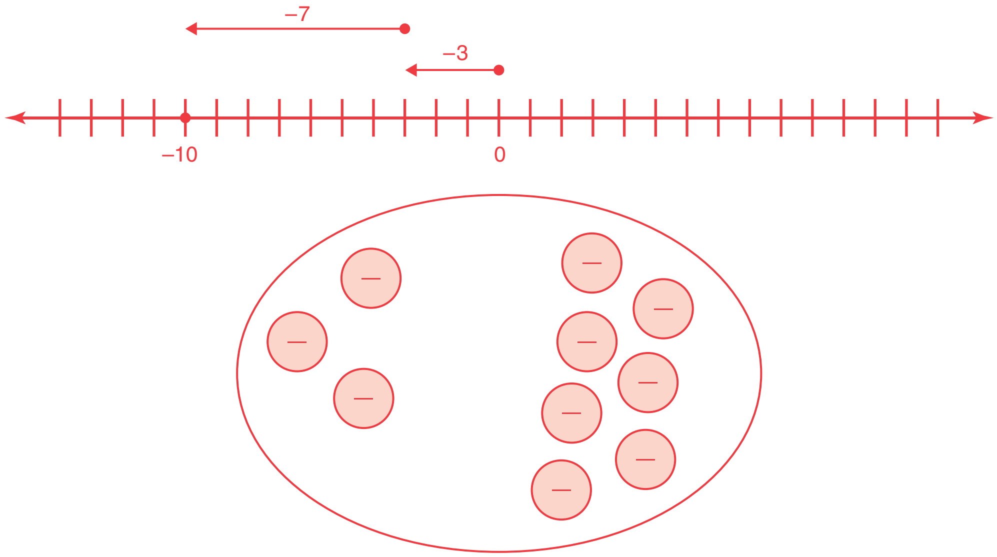
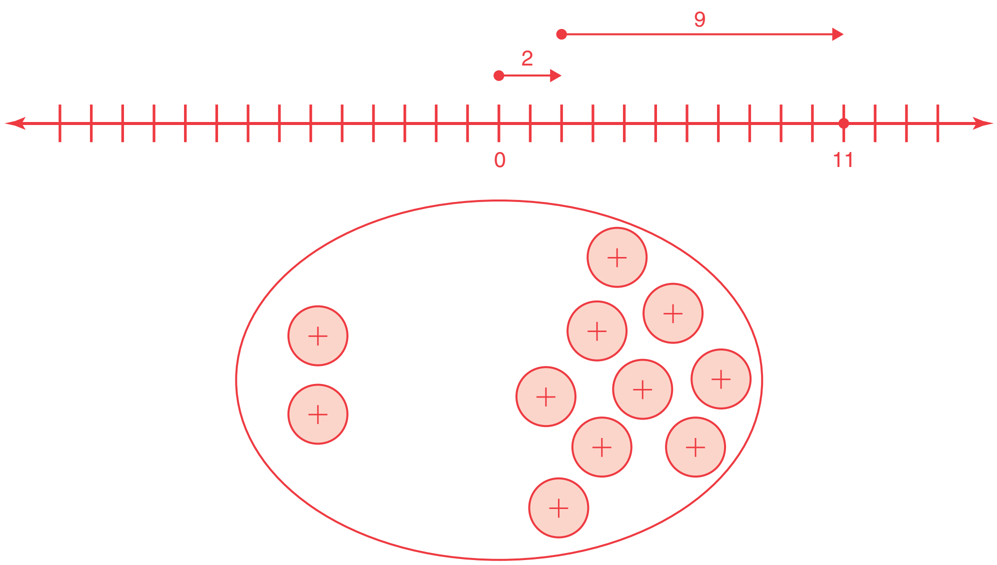
3. $(-3) + (-7) = -10$



4. $2 + 9 = 11$



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4.4 What's the Difference?

Subtracting Integers

Learning Goals

In this lesson, you will:

- Model subtraction of integers using two-color counters.
- Model subtraction of integers on a number line.
- Develop a rule for subtracting integers.

Key Term

- zero pair

Essential Ideas

- Subtraction can mean to take away objects from a set. Subtraction also describes the difference between two numbers.
- A zero pair is a pair of two-color counters composed of one positive counter (+) and one negative counter (−).
- Adding zero pairs to a two-color counter representation of an integer does not change the value of the integer.
- Subtraction of integers is modeled using two-color counters that represent positive charges (yellow counters) and negative charges (red counters).
- Subtraction of integers is modeled using a number line.
- Subtracting two negative integers is similar to adding two integers with opposite signs.
- Subtracting a positive integer from a positive integer is similar to adding two integers with opposite signs.
- Subtracting a positive integer from a negative integer is similar to adding two negative integers.
- Subtracting two integers is the same as adding the opposite of the number you are subtracting.

Common Core State Standards for Mathematics

7.NS The Number System

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
   a. Describe situations in which opposite quantities combine to make 0.
   b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

4

c. Understand subtraction of rational number as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

d. Apply properties of operations as strategies to add and subtract rational numbers.

Overview

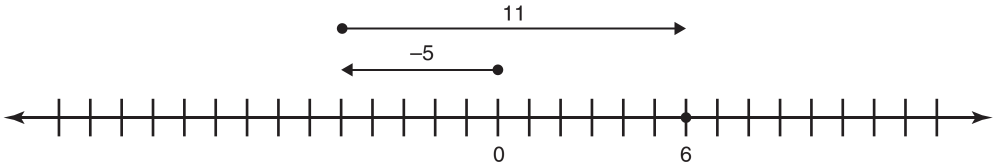
Two-color counters and number lines are used to model the difference of two integers. Through a series of activities, students will develop rules for subtracting integers. They conclude that subtracting two integers is the same as adding the opposite of the number you are subtracting. The first set of activities instructs students how to use zero pairs when performing subtraction using the two-color counter method. The term, zero pair is defined. Examples of modeling the difference between two integers with opposite signs using two-color counters is provided and the counters are paired together, one positive counter with one negative counter, until no possible pairs remain (the addition of zero pairs may or may not be needed). The resulting counters determine the difference of the integers. Students then draw a model for each given number sentence to determine the difference between two integers.

The number line method is used to model the difference between two integers. Students will conclude that subtracting two negative integers is similar to adding two integers with opposite signs, subtracting a positive integer from a positive integer is similar to adding two integers with opposite signs, and subtracting a positive integer from a negative integer is similar to adding two negative integers. Questions focus students to use algorithms to determine the difference of any two integers.

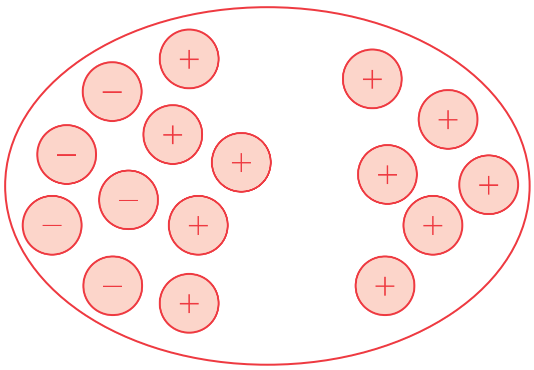
Warm Up

For each number line model, write the number sentence described by the model and draw a two-color counter model to represent the number sentence. Then, determine the sum.

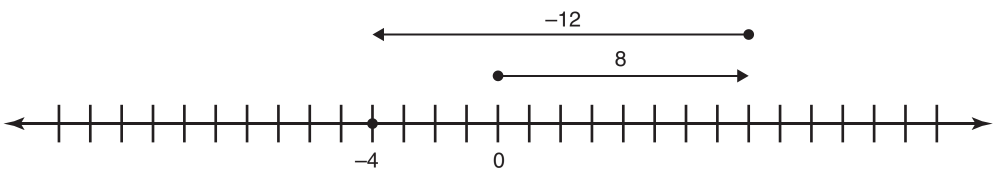
1.



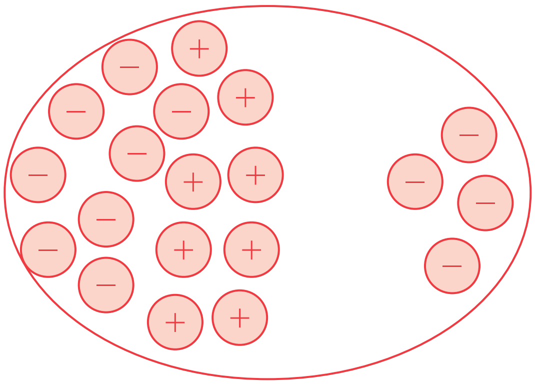
(−5) + 11 = 6



2.



8 + (−12) = −4



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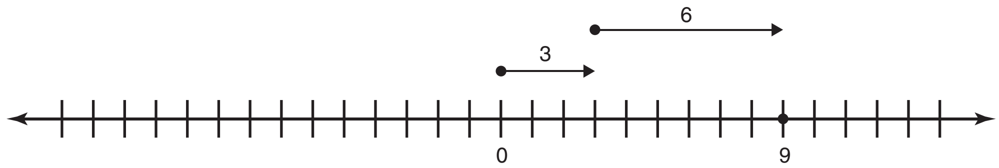
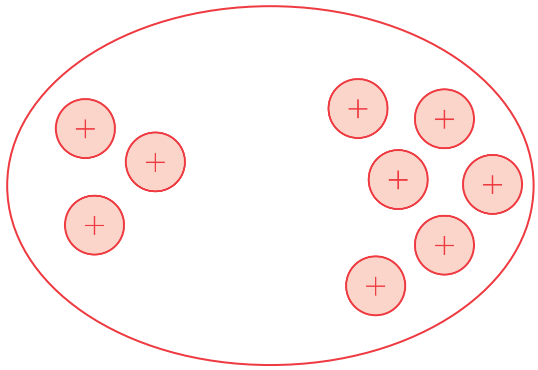
3.



$3 + 6 = 9$



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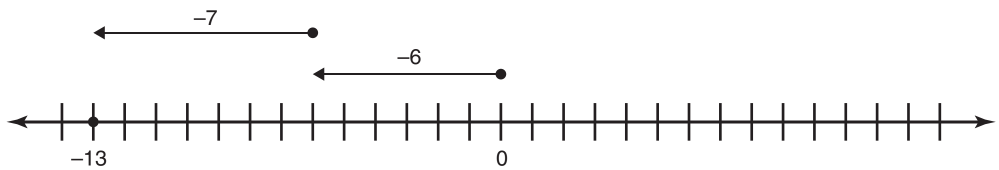
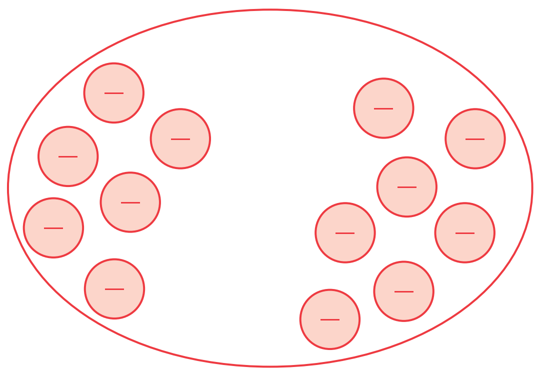


$(-6) + (-7) = -13$



4.4 What's the Difference?

Subtracting Integers

Learning Goals

In this lesson, you will:

- Model subtraction of integers using two-color counters.
- Model subtraction of integers on a number line.
- Develop a rule for subtracting integers.

Key Term

- zero pair

"I don't want nothing!" "We don't need no education." "I can't get no satisfaction." You may have heard—or even said—these phrases before. In proper English writing, however, these kinds of phrases should be avoided because they contain double negatives, which can make your writing confusing.

For example, the phrase "I don't need none" contains two "negatives": the word "don't" and the word "none." The sentence should be rewritten as "I don't need any." In mathematics, double negatives can be confusing as well, but it's perfectly okay to use them!

In this lesson, you will learn about subtracting integers, which sometimes involves double negatives.

4

Problem 1

Students complete a table by computing the difference between a maximum temperature and a minimum temperature for several states of the United States.

Grouping

Have students complete Questions 1 and 2 with a partner. Then share the responses as a class.

Share Phase, Question 1

- Which state has the highest maximum temperature?
- Which state has the lowest minimum temperature?
- Does the state having the highest maximum temperature also have the lowest minimum temperature?
- How did you compute the difference in temperatures for each state?
- If the difference between the maximum and minimum temperature for a particular state is very small, what does this tell you about the general climate of this state?
- If the difference between the maximum and minimum temperature for a particular state is very large, what does this tell you about the general climate of this state?

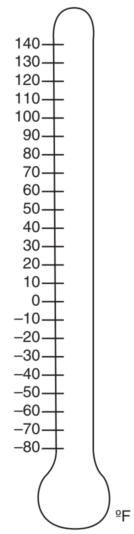
4

Problem 1 Temperatures

1. Complete the table to determine the difference between the maximum and minimum temperatures in each row.

United States Extreme Record Temperatures and Differences

State	Maximum Temp. (°F)	Minimum Temp. (°F)	Difference (°F)
Georgia	112	−17	129
Hawaii	100	12	88
Florida	109	−2	111
Alaska	100	−80	180
California	134	−35	169
North Carolina	110	−34	144
Arizona	128	−40	168
Texas	120	−23	143



a. Which state shows the least difference between the maximum and minimum temperature?

Hawaii has the least difference in high and low extremes with 88°F.

b. Which state shows the greatest difference between the maximum and minimum temperature?

Alaska has the greatest difference in high and low extremes with 180°F.

Problem 2

The two-color counter model is used to compute the difference between two integers. Zero pairs are introduced to support subtracting a larger integer from a smaller integer. Examples of using this model are provided and students will complete partially drawn models. They then create a model for a number sentence that describes a subtraction problem and calculate the difference. Finally, students will write a rule for subtracting positive and negative integers.

Grouping

Ask a student to read the introduction to Problem 2 aloud. Discuss the worked examples and complete Questions 1 and 2 as a class.

2. You overheard a radio announcer report that from 12:00 PM to 3:00 PM the temperature went from -5°F to -10°F. He said, "It is getting warmer." Was he correct? Explain your reasoning.

The radio announcer was not correct. The temperature dropped 5°F.

Problem 2 Models for Subtracting Integers

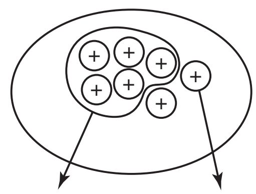
Subtraction can mean to "take away" objects from a set. Subtraction can also mean a comparison of two numbers, or the "difference between them."

The number line model and the two-color counter model used in the addition of integers can also be used to investigate the subtraction of integers.

Using just positive or just negative counters, you can show subtraction using the "take away" model.

Example 1: $^{+}7 - {}^{+}5$

First, start with seven positive counters.

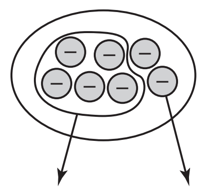


Then, take away five positive counters. Two positive counters remain.

$^{+}7 - {}^{+}5 = {}^{+}2$

Example 2: $-7 - (-5)$

First, start with seven negative counters.



Then, take away five negative counters. Two negative counters remain.

$-7 - (-5) = -2$

4

1. How are Examples 1 and 2 similar? How are these examples different?

   Both examples show subtracting integers with the same sign. Example 1 shows the difference between two positive integers and Example 2 shows the difference between two negative integers.

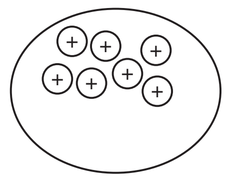
To subtract integers using both positive and negative counters, you will need to use *zero pairs*.

$$\oplus + \ominus = 0$$

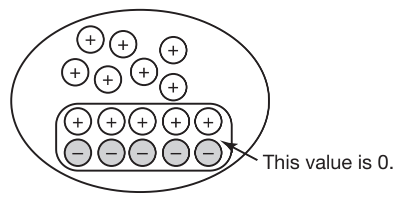
Recall that the value of a $\ominus$ and $\oplus$ pair is zero. So, together they form a **zero pair**. You can add as many pairs as you need and not change the value.

Example 3: $^{+}7 - (-5)$

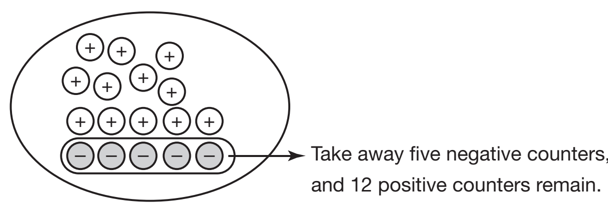
Start with seven positive counters.



The expression says to subtract five negative counters, but there are no negative counters in the first model. Insert five negative counters into the model. So that you don't change the value, you must also insert five positive counters.



Now, you can subtract, or take away, the five negative counters.



$$^{+}7 - (-5) = {}^{+}12$$

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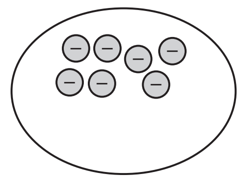
Example 4: $-7 - {}^{+}5$

Start with seven negative counters.

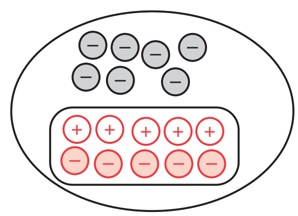


2. The expression says to subtract five positive counters, but there are no positive counters in the first model.

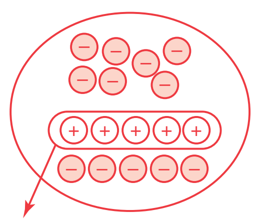
   a. How can you insert positive counters into the model and not change the value?

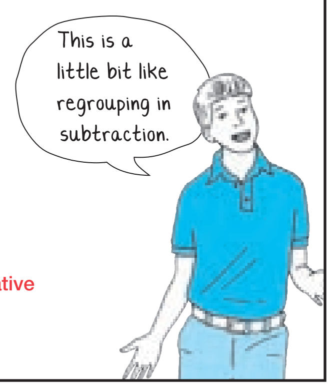
   I can insert five positive counters and five negative counters and not change the value.

   b. Complete the model.



   c. Now, subtract, or take away, the five positive counters. Sketch the model to show that $-7 - {}^{+}5 = -12$.





Remove the five positive counters, and 12 negative counters remain.

Share Phase, Question 2

- How do you know how many counters to start with, in the model?
- How do you know how many counters to take away, in the model?
- How do you know if you need to add zero pairs to the model?
- How many zero pairs can be added to the model?
- Is it possible to add too many zero pairs to the model? Explain.
- Is it possible to not add enough zero pairs to the model? Explain.
- What happens if you do not add enough zero pairs to the model?
- What happens if you add more zero pairs than you need to the model?
- How is 'taking away' denoted in the model?
- Where is the answer or the difference between the two integers in the model?

4

Grouping

Have students complete Questions 3 through 6 with a partner. Then share the responses as a class.

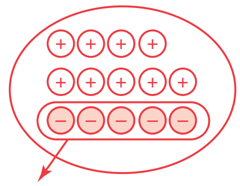
Share Phase, Question 3

- What is the difference between a subtraction sign and a negative sign?
- Is there a difference between the integer 4 and −4? Or are they the same integer?
- Explain how the integers 4 and −4 are different, and how they are alike.

3. Draw a representation for each subtraction problem. Then, calculate the difference.

a. $4 - (-5)$

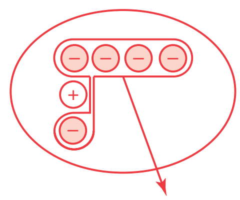
$= 9$



Start with four positive counters, and add five zero pairs. Then, subtract five negative counters. The result is nine positive counters.

b. $-4 - (-5)$

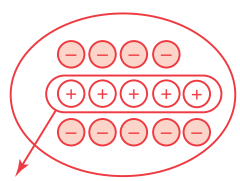
$= 1$



Start with four negative counters, and add one zero pair. Then, subtract five negative counters. The result is one positive counter.

c. $-4 - {}^{+}5$

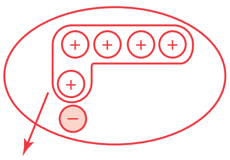
$= -9$



Start with four negative counters, and add five zero pairs. Then, subtract five positive counters. The result is nine negative counters.

d. $4 - 5$

$= -1$



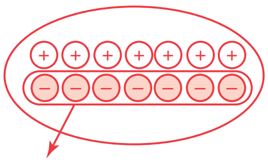
Start with four positive counters, and add one zero pair. Then, subtract five positive counters. The result is one negative counter.

Share Phase, Question 4

- Can this type of model be used to compute the difference for any subtraction problem?
- What is an example of a subtraction problem that would not be easily solved using this model? Explain.

4. How could you model $0 - (-7)$?

a. Draw a sketch of your model. Finally, determine the difference.



Start with 0, and add seven zero pairs. Then, subtract seven negative counters. The result is seven positive counters.

b. In part (a), would it matter how many zero pairs you add? Explain your reasoning.

It would not matter how many zero pairs I add. Once I remove the seven negative counters and have seven remaining positive counters, it does not matter how many additional pairs of positive and negative counters are left because their value is zero.

4

Share Phase, Questions 5 and 6

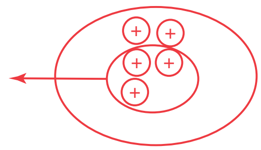
- Can a subtraction problem have more than one correct answer?
- Can more than one subtraction problem give you the same answer?
- Is there another way to write this rule?

5. Does the order in which you subtract two numbers matter? Does $5 - 3$ have the same answer as $3 - 5$? Draw models to explain your reasoning.

   Subtraction is not commutative, so the order matters.

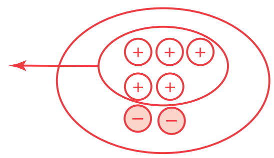
   $5 - 3 = 2$

   In this expression, I would start with five positive counters and subtract three. The result is two positive counters.

   $3 - 5 = -2$

   In this expression, I would start with three positive counters and add two zero pairs. Then, I would subtract five positive counters. The result is two negative counters.

6. Write a rule for subtracting positive and negative integers.

   Answers will vary.

4

Problem 3

The number line model is used to compute the difference between two integers. Examples of using this model are provided and students explain the drawn models. They will then create a model for a number sentence that describes a subtraction problem and calculate the difference. Next, students analyze number sentences to look for patterns. They will determine unknown integers in number sentences and compute absolute values of differences.

Grouping

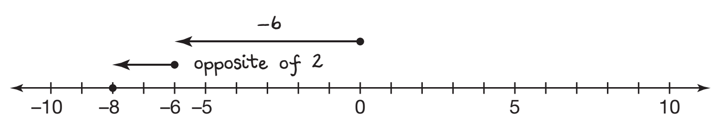
Have students complete Questions 1 and 2 with a partner. Then share the responses as a class.

Problem 3 Subtracting on a Number Line

Cara thought of subtraction of integers another way. She said, "Subtraction means to back up, or move in the opposite direction. Like in football when a team is penalized or loses yardage, they have to move back."

Analyze Cara's examples.

Example 1: $-6 - (+2)$



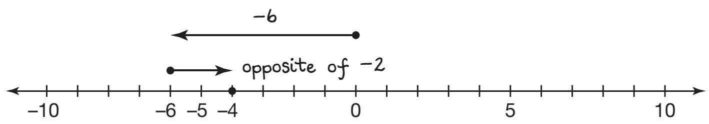
First, I moved from zero to −6, and then I went in the opposite direction of the +2 because I am subtracting. So, I went two units to the left and ended up at −8.

$-6 - (+2) = -8$

Share Phase, Questions 1 and 2

- How is Example 1 similar to Example 2?
- How is Example 1 different from Example 2?
- How is Example 3 similar Example 4?
- How is Example 3 different from Example 4?
- What do all four examples have in common?

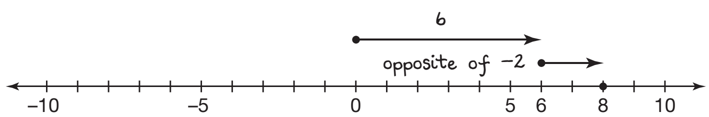
Example 2: $-6 - (-2)$



In this problem, I went from zero to −6. Because I am subtracting (−2), I went in the opposite direction of the −2, or right two units, and ended up at −4.

$-6 - (-2) = -4$

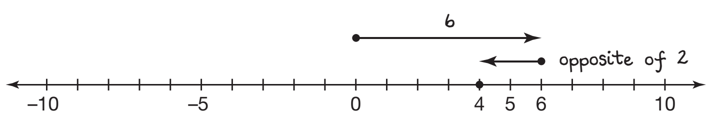
Example 3: $6 - (-2)$



1. Explain the model Cara created in Example 3.

   Cara went from 0 to 6. Because the problem says to subtract (−2), she went in the opposite direction of (−2), or to the right two units, and ended at 8.

Example 4: $6 - (+2)$



2. Explain the model Cara created in Example 4.

   Cara went from 0 to 6. Because the problem says to subtract (+2), she went in the opposite direction of (+2), or to the left two units, and ended at 4.

4

Grouping

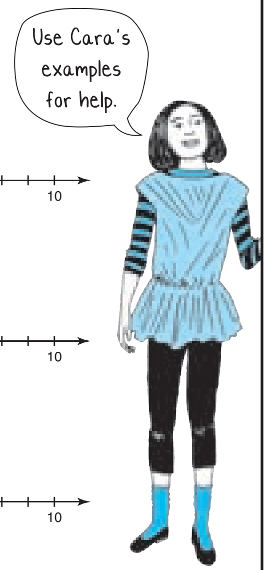
Have students complete Questions 3 and 4 with a partner. Then share the responses as a class.

Share Phase, Questions 3 and 4

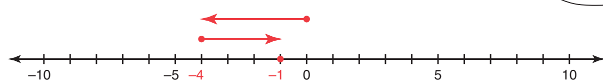
- What is the first step toward solving this problem?
- Is there more than one way to begin solving this problem?
- Is the first step the same for a subtraction problem and an addition problem when using the number line model?
- What is the second step toward solving this problem?
- How is the second step different for subtraction, when comparing it to the second step you used when computing the sum of two integers, with respect to the number line model?
- How do you know if the arrows should be pointing in different directions when using the number line model?
- How do you know if the arrows should be pointing in the same direction when using the number line model?
- If the difference between the two integers is zero, how would you describe the arrows on the number line model?

4

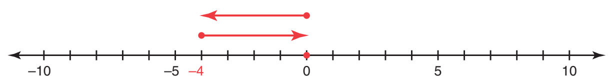
3. Use the number line to complete each number sentence.



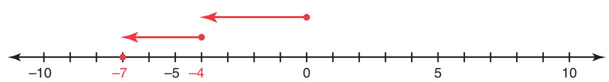
a. $-4 - (-3) =$ <u>−1</u>



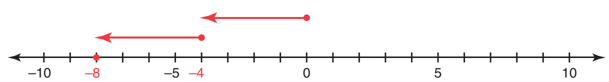
b. $-4 - (-4) =$ <u>0</u>



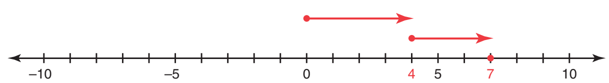
c. $-4 - {}^{+}3 =$ <u>−7</u>



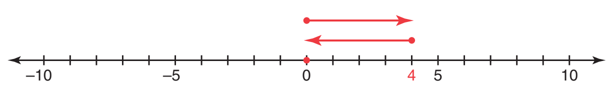
d. $-4 - {}^{+}4 =$ <u>−8</u>



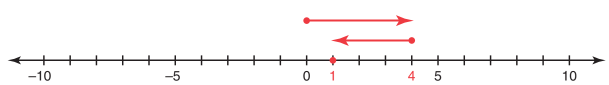
e. ${}^{+}4 - (-3) =$ <u>7</u>



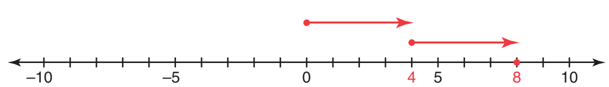
f. ${}^{+}4 - {}^{+}4 =$ <u>0</u>



g. ${}^{+}4 - {}^{+}3 =$ <u>1</u>



h. ${}^{+}4 - (-4) =$ <u>8</u>



Grouping

- Have students complete Question 5 with a partner. Then share the responses as a class.
- Ask a student to read the information following Question 5 aloud. Discuss the worked example as a class.

Share Phase, Question 5

- Using a number line, if you begin at −13 and go to −12, would this be considered an increase or a decrease?
- Using a number line, if you begin at −12 and go to −13, would this be considered an increase or a decrease?

4. What patterns did you notice when subtracting the integers in Question 3?

a. Subtracting two negative integers is similar to

adding two integers with opposite signs.

b. Subtracting two positive integers is similar to

adding two integers with opposite signs.

c. Subtracting a positive integer from a negative integer is similar to

adding two negative integers.

d. Subtracting a negative integer from a positive integer is similar to

adding two positive integers.

5. Analyze the number sentences shown.

- $-8 - 5 = -13$
- $-8 - 3 = -11$
- $-8 - 1 = -9$
- $-8 - 4 = -12$
- $-8 - 2 = -10$
- $-8 - 0 = -8$

a. What patterns do you see? What happens as the integer subtracted from −8 decreases?

As the integer subtracted from 8 decreases, the result increases.

b. From your pattern, predict the answer to $-8 - (-1)$.

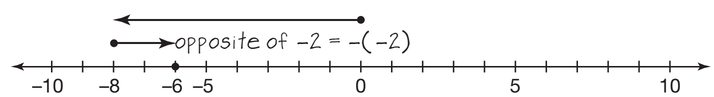
The answer is −7.

For a subtraction expression, such as $-8 - (-2)$, Cara's method is to start at zero and go to −8, and then go two spaces in the opposite direction of −2 to get −6.

Dava says, "I see another pattern. Since subtraction is the inverse of addition, you can think of subtraction as adding the opposite number. That matches with Cara's method of going in the opposite direction."

$-8 - (-2)$ is the same as $-8 + -(-2)$

$-8 + 2 = -6$



Grouping

- Have students complete Question 6 with a partner. Then share the responses as a class.
- Have students complete Questions 7 through 10 with a partner. Then share the responses as a class.

Share Phase, Question 6

- When you subtract a negative number from another number, is this the same as adding the number?
- How is subtracting a negative number similar to adding the number?
- Did you need a two-color counter model or a number line model to compute the difference?
- Why would it be impractical to use a two-color counter model to compute the difference?
- Why would it be impractical to use a number line model to compute the difference?

Share Phase, Question 7

- How did you determine the unknown integer?
- Can you tell the sign of the unknown integer by looking at the problem?
- How can you tell the sign of the unknown integer by looking at the problem?

4

An example of Dava's method is shown.

$^{+}10 - (-4) =$
$10 + -(-4)$
$10 + 4 = 14$

6. Apply Dava's method to determine each difference.

a. $-9 - (-2) =$
$-9 + 2 = -7$

b. $-3 - (-3) =$
$-3 + 3 = 0$

c. $-7 - {}^{+}5 =$
$-7 + (-5) = -12$

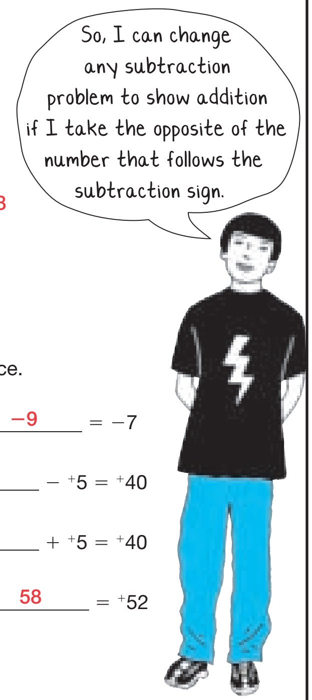
d. $^{+}24 - {}^{+}8 =$
$24 + (-8) = 16$

e. $-4 - {}^{+}2 =$
$-4 + (-2) = -6$

f. $^{+}5 - {}^{+}9 =$
$5 + (-9) = -4$

g. $-20 - (-30) =$
$-20 + 30 = 10$

h. $-10 - {}^{+}18 =$
$-10 + (-18) = -28$



7. Determine the unknown integer in each number sentence.

a. $^{+}3 +$ __4__ $= {}^{+}7$

b. $^{+}2 +$ __−9__ $= -7$

c. __10__ $+ -20 = -10$

d. __45__ $- {}^{+}5 = {}^{+}40$

e. __35__ $- (-5) = {}^{+}40$

f. __35__ $+ {}^{+}5 = {}^{+}40$

g. $^{+}6 +$ __46__ $= {}^{+}52$

h. $-6 +$ __58__ $= {}^{+}52$

i. $-6 +$ __−46__ $= -52$

- Under what circumstance is the sign of the unknown integer negative?
- Under what circumstance is the sign of the unknown integer positive?
- At a glance, can you tell if the difference between two integers is positive?
- At a glance, can you tell if the difference between two integers is negative?
- At a glance, can you tell if the difference between two integers is zero?

Share Phase, Questions 8 through 10

- How are the operations of addition and subtraction similar?
- How are the operations of addition and subtraction different?
- Why are addition and subtraction thought of as opposite operations?
- Does addition 'undo' subtraction? How?
- Does subtraction 'undo' addition? How?

Talk the Talk

Students decide whether subtraction sentences are always true, sometimes true, or never true and use examples to justify their reasoning. Several questions focus students on the relationships between integer addition and subtraction.

Grouping

Have students complete Questions 1 through 6 with a partner. Then share the responses as a class.

8. Determine each absolute value.

a. $|-7 - (-3)|$ 4

b. $|-7 - {}^{+}3|$ 10

c. $|7 - {}^{+}3|$ 4

d. $|7 - (-3)|$ 10

9. How does the absolute value relate to the distance between the two numbers in Question 8, parts (a) through (d)?

The absolute value of each expression is the same as the number of units, or the distance, between the two numbers if graphed on a number line.

10. Is $|8 - 6|$ equal to $|6 - 8|$? Is $|4 - 6|$ equal to $|6 - 4|$? Explain your thinking.

$|8 - 6|$ is equal to $|6 - 8|$.

$|4 - 6|$ is equal to $|6 - 4|$.

The absolute values are the same because the distance between the two numbers is the same.

Talk the Talk

1. Tell whether these subtraction sentences are always true, sometimes true, or never true. Give examples to explain your thinking.

a. positive − positive = positive

Sometimes true. $10 - 4 = 6$ but $4 - 8 = -4$

b. negative − positive = negative

Always true. $-10 - 4 = -14$

c. positive − negative = negative

Never true. $10 - (-2) = 12$ or $2 - (-10) = 12$

d. negative − negative = negative

Sometimes true. $-5 - (-2) = -3$ but $-2 - (-5) = 3$

4

2. If you subtract two negative integers, will the answer be greater than or less than the number you started with? Explain your thinking.

The answer will be greater than the number you started with. For example, $-10 - (-1) = -9$ and -9 is greater than -10. $-10 - (-21) = 11$ and 11 is greater than -10.

3. What happens when a positive number is subtracted from zero?

The result will be the opposite of the number you subtracted from zero. It will be the negative of that number.

4. What happens when a negative number is subtracted from zero?

The result will be the opposite of the number you subtracted from zero. It will be the positive of that number.

5. Just by looking at the problem, how do you know if the sum of two integers is positive, negative, or zero?

If both integers are positive, then the result is positive. If both numbers are negative, then the result is negative. If the numbers are opposites, then the result is zero. If you are adding two integers with different signs, then the sign of the number with the greater absolute value determines the sign of the result.

6. How are addition and subtraction of integers related?

Subtracting two integers is the same as adding the opposite of the number you are subtracting.

Be prepared to share your solutions and methods.

4

Follow Up

Assignment

Use the Assignment for Lesson 4.4 in the Student Assignments book. See the Teacher's Resources and Assessments book for answers.

Skills Practice

Refer to the Skills Practice worksheet for Lesson 4.4 in the Student Assignments book for additional resources. See the Teacher's Resources and Assessments book for answers.

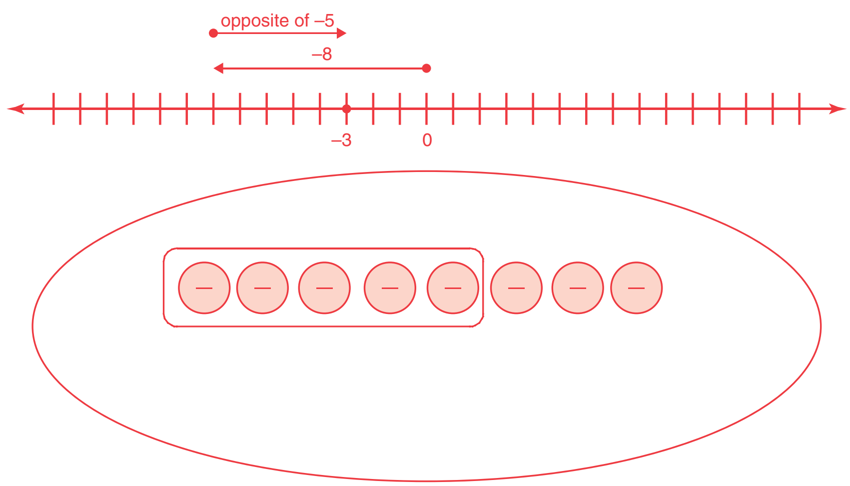
Assessment

See the Assessments provided in the Teacher's Resources and Assessments book for Chapter 4.

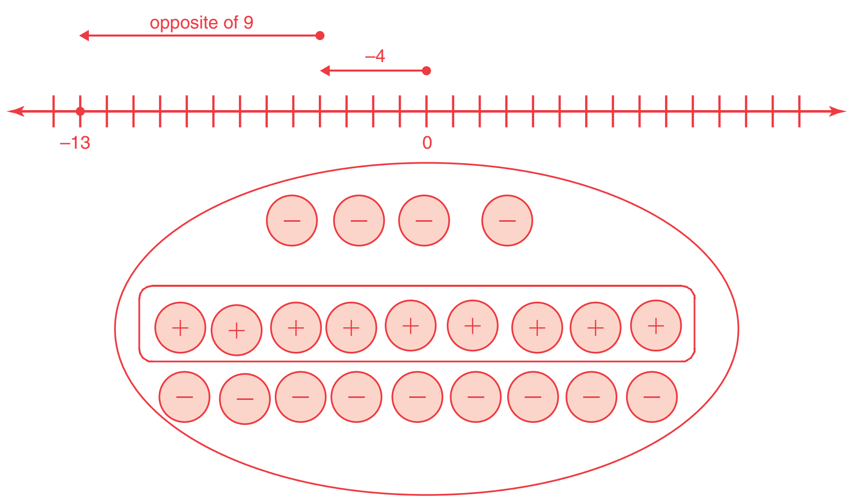
Check for Students' Understanding

Draw both, a model using two-color counters, and a model using a number line, to represent each number sentence. Then, determine the difference.

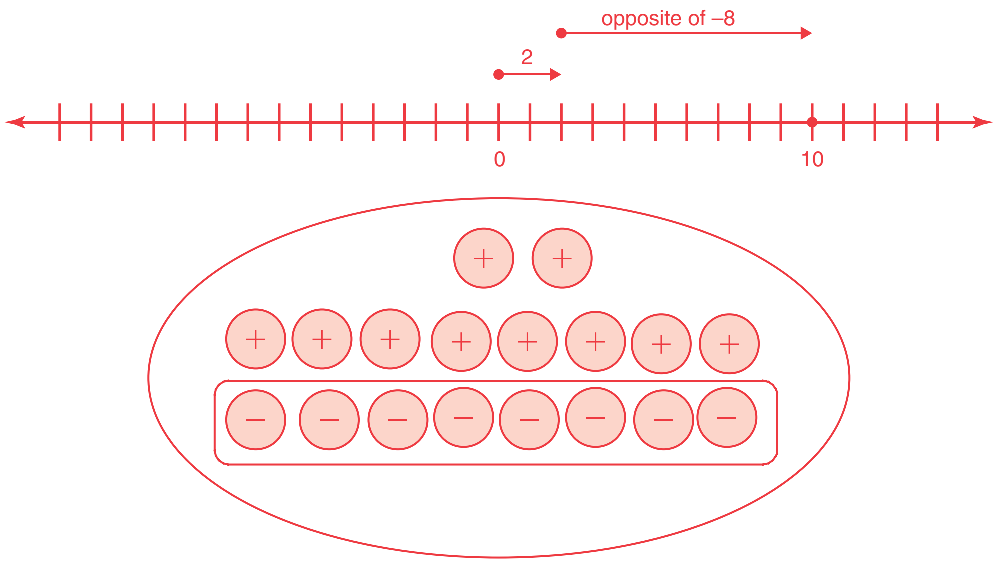
1. $-8 - (-5) = -3$



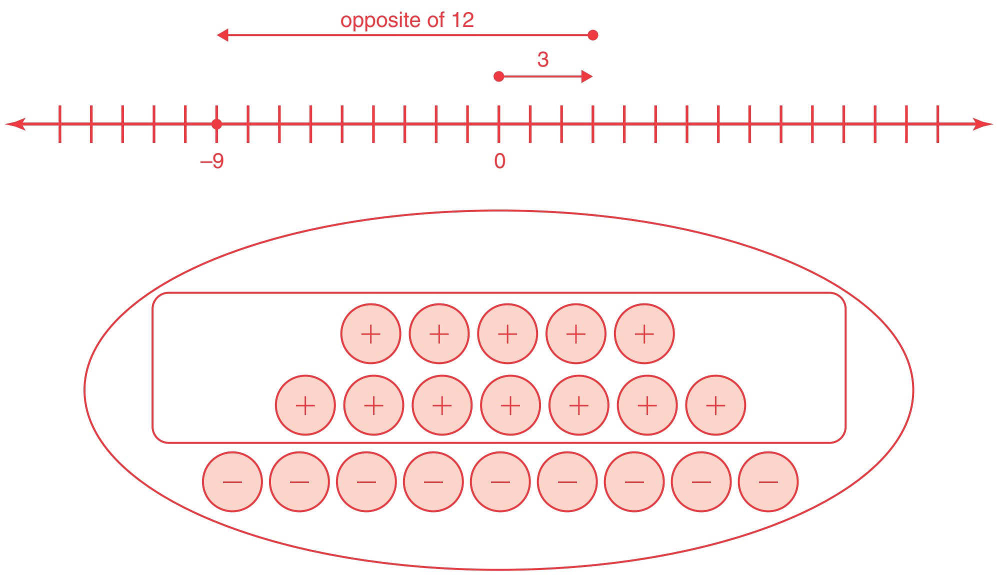
2. $-4 - {}^{+}9 = -13$



3. $2 - (-8) = 10$



4. $3 - {}^{+}12 = -9$



4

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4.5 What Do We Do Now?

Adding and Subtracting Rational Numbers

Learning Goal

In this lesson, you will:

- Add and subtract rational numbers.

Essential Idea

- The rules for combining integers also apply to combining rational numbers.

Common Core State Standards for Mathematics

7.NS The Number System

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

c. Understand subtraction of rational number as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

d. Apply properties of operations as strategies to add and subtract rational numbers.

4

Overview

Students apply their knowledge of adding and subtracting positive and negative integers to the set of rational numbers.

4

Warm Up

Determine each sum.

1. $\frac{1}{6} + \frac{1}{3}$

$\frac{1}{6} + \frac{1}{3} = \frac{1}{6} + \frac{2}{6} = \frac{3}{6}$

2. $\frac{2}{7} + \frac{2}{5}$

$\frac{2}{7} + \frac{2}{5} = \frac{10}{35} + \frac{14}{35} = \frac{24}{35}$

3. $\frac{1}{2} + \frac{3}{5}$

$\frac{1}{2} + \frac{3}{5} = \frac{5}{10} + \frac{6}{10} = \frac{11}{10} = 1\frac{1}{10}$

4. $\frac{1}{3} + \frac{4}{5}$

$\frac{1}{3} + \frac{4}{5} = \frac{5}{15} + \frac{12}{15} = \frac{17}{15} = 1\frac{2}{15}$

4

4

4.5 WHAT DO WE DO NOW?

Adding and Subtracting Rational Numbers

Learning Goal

In this lesson, you will:

- Add and subtract rational numbers.

You might think that as you go deeper below the Earth's surface, it would get colder. But this is not the case.

Drill down past the Earth's crust, and you reach a layer called the mantle, which extends to a depth of about -1800 miles. The temperature in this region is approximately $+1600$°F. Next stop is the outer core, which extends to a depth of about -3200 miles and has a temperature of approximately $+8000$°F. The last stop is the very center, the inner core. At approximately -4000 miles, the inner core may have a temperature as high as 12,000°F—as high as the temperature on the surface of the Sun!

What do you think makes the temperature increase as elevation decreases?

4

Problem 1

The problem $-3\frac{3}{4} + 4\frac{1}{4}$ is solved using a number line and a two-color counter model. Students describe each method in their own words and interpret the answer. Then larger mixed numbers are used to formulate another problem and students explain why using the two methods would not be practical. Questions focus students on using the familiar rules of combining integers from the previous lesson to solve problems involving the computation of the sum of two mixed numbers and the sum of two decimals.

Grouping

Ask a student to read the introduction to Problem 1 aloud. Discuss the peer analysis and complete Question 1 as a class.

Discuss Phase, Question 1

- Where did Kaitlin begin on the number line?
- What direction did Kaitlin move first on the number line? Why?
- What direction did Kaitlin move second on the number line? Why?
- Where did Kaitlin end up on the number line?
- What is Kaitlin's answer?
- What do the darker shapes represent in Omar's model?
- What do the lighter shapes represent in Omar's model?

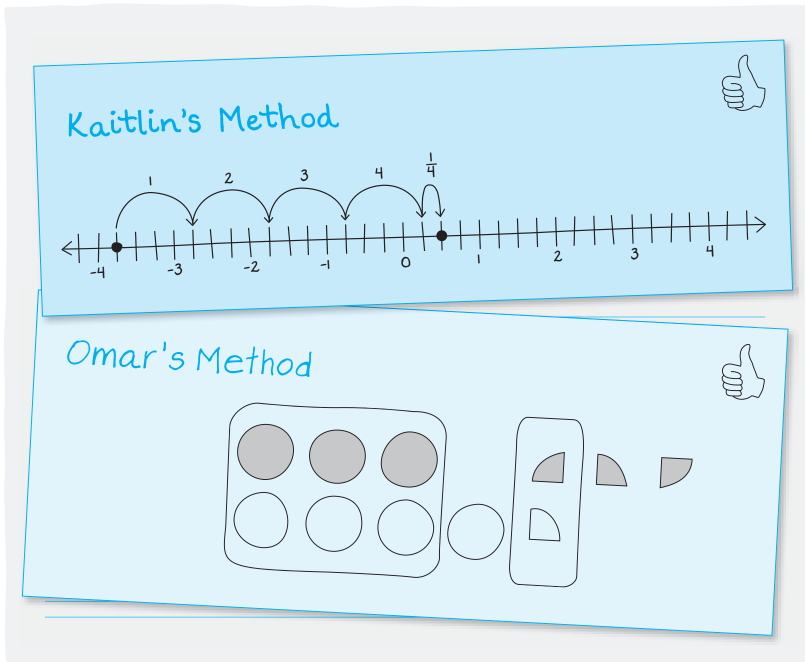
4

Problem 1 Adding Rational Numbers

Previously, you learned how to add and subtract with positive and negative integers. In this lesson, you will apply what you know about your work with integers to the set of rational numbers.

Consider this problem and the two methods shown.

$$-3\frac{3}{4} + 4\frac{1}{4} = ?$$



1. Describe each method and the correct answer.

   Kaitlin used a number line model. She began at $-3\frac{3}{4}$ and moved to the right $4\frac{1}{4}$ to get the answer $\frac{1}{2}$.
   Omar used two-color counters. First, he modeled $-3\frac{3}{4}$ and $4\frac{1}{4}$. Then, he circled the zero pairs, and had 1 whole positive counter and two $\frac{1}{4}$ negative counters remaining. So, the answer is $\frac{1}{2}$.

- What does one dark circle represent in Omar's model?
- What does one white circle represent in Omar's model?
- What does each part of a circle represent in Omar's model?
- What is Omar's answer?

Grouping

Have students complete Questions 2 and 3 with a partner. Then share the responses as a class.

Share Phase, Questions 2 and 3

- How is this problem different from the last problem?
- At what point are the numbers too large to use the number line model?
- At what point are the numbers too large to use the two-color counter model?
- At what point are the numbers too small to use the number line model?
- At what point are the numbers too small to use the two-color counter model?
- Can the rules you learned for combining integers be applied to combining mixed numbers? How?
- To combine the two mixed numbers, do you need a common denominator? Explain.
- How do you determine the common denominator needed to combine the mixed numbers?
- What is the common denominator needed to combine the mixed numbers?

2. Now, consider this problem:

$$12\frac{1}{3} + \left(-23\frac{3}{4}\right) = ?$$

   a. Why might it be difficult to use either a number line or counters to solve this problem?

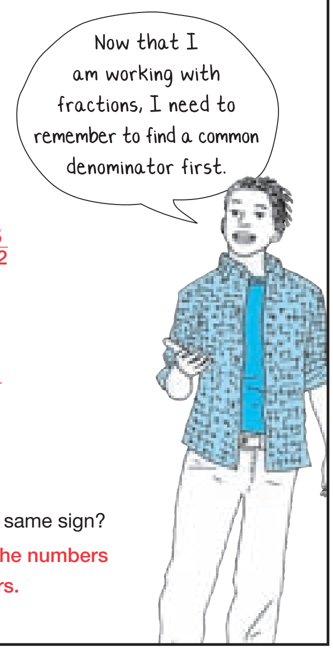
   I would need to draw a really long number line or a lot of counters.

   b. What is the rule for adding signed numbers with different signs?

   If you are adding two numbers with different signs, then the sign of the number with the greater absolute value determines the sign of the result, and you subtract the number with the smaller absolute value from the number with the larger absolute value.

   c. What will be the sign of the sum for this problem? Explain your reasoning.

   The answer will be negative because the absolute value of the negative number is greater.



   d. Calculate the sum.

$$12\frac{1}{3} + \left(-23\frac{3}{4}\right) = -11\frac{5}{12}$$

$$\begin{array}{r} 23\frac{3}{4} = 23\frac{9}{12} \\ -\,12\frac{1}{3} = 12\frac{4}{12} \\ \hline 11\frac{5}{12} \end{array}$$

3. What is the rule for adding signed numbers with the same sign?

   If the numbers have the same sign, then you add the numbers and the sign of the sum is the same as the numbers.

4

Grouping

Have students complete Question 4 with a partner. Then share the responses as a class.

Share Phase, Question 4

- How do you determine the common denominator needed to combine the mixed numbers?
- Do we have to use the least common denominator to solve this problem? Explain.
- What is the common denominator needed to combine the mixed numbers?
- At a glance, can you determine the sign of the answer? How?
- Is it easier to combine two mixed numbers or two decimals? Why?

4. Determine each sum. Show your work.

a. $-5\frac{3}{5} + 6\frac{1}{3} =$

$-5\frac{3}{5} + 6\frac{1}{3} = \frac{11}{15}$

$$\begin{array}{rl} 6\frac{1}{3} = 6\frac{5}{15} & = 5\frac{20}{15} \\ -\ 5\frac{3}{5} & = 5\frac{9}{15} \\ \hline & \frac{11}{15} \end{array}$$

b. $-3\frac{2}{3} + \left(-4\frac{2}{3}\right) =$

$-3\frac{2}{3} + \left(-4\frac{2}{3}\right) = -8\frac{1}{3}$

$$\begin{array}{r} 3\frac{2}{3} \\ +\ 4\frac{2}{3} \\ \hline 7\frac{4}{3} = 8\frac{1}{3} \end{array}$$

c. $-7.34 + 10.6 =$

$-7.34 + 10.6 = 3.26$

$$\begin{array}{r} \overset{5\,10}{10.\cancel{6}\cancel{0}} \\ -7.34 \\ \hline 3.26 \end{array}$$

d. $17\frac{2}{3} + 11\frac{1}{6} =$

$17\frac{2}{3} + 11\frac{1}{6} = 28\frac{5}{6}$

$$\begin{array}{rl} 17\frac{2}{3} & = 17\frac{4}{6} \\ +\ 11\frac{1}{6} & = 11\frac{1}{6} \\ \hline & 28\frac{5}{6} \end{array}$$

e. $-104\frac{3}{4} + 88\frac{1}{6} =$

$-104\frac{3}{4} + 88\frac{1}{6} = -16\frac{7}{12}$

$$\begin{array}{rl} 104\frac{3}{4} & = 104\frac{9}{12} \\ -\ 88\frac{1}{6} & = 88\frac{2}{12} \\ \hline & 16\frac{7}{12} \end{array}$$

f. $-27 + 16.127 =$

$-27 + 16.127 = -10.873$

$$\begin{array}{r} 2\overset{6}{\cancel{7}}.\overset{9}{\cancel{0}}\overset{9}{\cancel{0}}\overset{10}{\cancel{0}} \\ -\ 16.127 \\ \hline 10.873 \end{array}$$

Problem 2

Questions focus students on using the familiar rules of combining integers from the previous lesson to solve problems involving the computation of the difference between two mixed numbers and the difference between two decimals.

Grouping

Have students complete Questions 1 and 2 with a partner. Then share the responses as a class.

Share Phase, Questions 1 and 2

- How do you determine the common denominator needed to determine the difference between two mixed numbers?
- Do we have to use the least common denominator to solve this problem? Explain.
- What is the common denominator needed to compute the difference between the two mixed numbers?
- At a glance, can you determine the sign of the answer? How?
- Is it easier to compute the difference between two mixed numbers or the difference between two decimals? Why?

Problem 2 Subtracting Rational Numbers

1. What is the rule for subtracting signed numbers?

   Subtracting two numbers is the same as adding the opposite of the number you are subtracting.

2. Determine each difference. Show your work.

   a. $-5\frac{1}{5} - 6\frac{2}{3} =$

   $-5\frac{1}{5} - 6\frac{2}{3} = -5\frac{1}{5} + \left(-6\frac{2}{3}\right) = -11\frac{13}{15}$

   $$\begin{array}{r} 5\frac{1}{5} = 5\frac{3}{15} \\ +\ 6\frac{2}{3} = 6\frac{10}{15} \\ \hline 11\frac{13}{15} \end{array}$$

   b. $8\frac{1}{4} - \left(-5\frac{1}{3}\right) =$

   $8\frac{1}{4} - \left(-5\frac{1}{3}\right) = 8\frac{1}{4} + 5\frac{1}{3} = 13\frac{7}{12}$

   $$\begin{array}{r} 8\frac{1}{4} = 8\frac{3}{12} \\ +\ 5\frac{1}{3} = 5\frac{4}{12} \\ \hline 13\frac{7}{12} \end{array}$$

   c. $-7\frac{3}{4} - \left(-4\frac{7}{8}\right) =$

   $-7\frac{3}{4} - \left(-4\frac{7}{8}\right) = -7\frac{3}{4} + 4\frac{7}{8} = -2\frac{7}{8}$

   $$\begin{array}{r} 7\frac{3}{4} = 7\frac{6}{8} = 6\frac{14}{8} \\ -\ 4\frac{7}{8} = \phantom{7\frac{6}{8} =}\ 4\frac{7}{8} \\ \hline 2\frac{7}{8} \end{array}$$

   d. $-11\frac{1}{2} - 12\frac{1}{5} =$

   $-11\frac{1}{2} - 12\frac{1}{5} = -11\frac{1}{2} + \left(-12\frac{1}{5}\right) = -23\frac{7}{10}$

   $$\begin{array}{r} 11\frac{1}{2} = 11\frac{5}{10} \\ +\ 12\frac{1}{5} = 12\frac{2}{10} \\ \hline 23\frac{7}{10} \end{array}$$

   e. $-24.15 - (13.7) =$

   $-24.15 - (13.7) = -37.85$

   $$\begin{array}{r} 24.15 \\ +\ 13.70 \\ \hline 37.85 \end{array}$$

   f. $-6.775 - (-1.7) =$

   $-6.775 - (-1.7) = -5.075$

   $$\begin{array}{r} 6.775 \\ -\ 1.700 \\ \hline 5.075 \end{array}$$

4

Problem 3

Students add or subtract two rational numbers using the algorithms they wrote in the previous problem.

Grouping

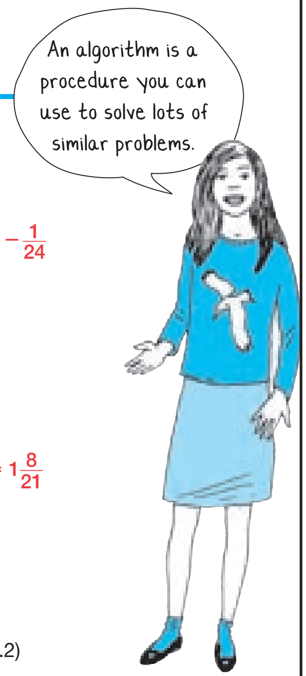
Have students complete Questions 1 through 20 with a partner. Then share the responses as a class.

Share Phase, Questions 1 through 20

- What is an algorithm?
- Is an algorithm the same as a rule?
- Can these problems be rewritten vertically?
- Is it easier to solve the problem if it's written vertically or horizontally? Why?
- Is it easier to use an algorithm, the two-counter model, or the number line model to compute the difference between two rational numbers? Explain.
- At a glance, can you tell if the answer to the problem will be greater than zero, less than zero, or equal to zero? Explain.

4

Problem 3 Adding and Subtracting with an Algorithm



Add and subtract using your algorithms.

1. $4.7 + -3.65$

   1.05

2. $-\frac{2}{3} + \frac{5}{8}$

   $-\frac{16}{24} + \frac{15}{24} = -\frac{1}{24}$

3. $3.95 + -6.792$

   -2.842

4. $2\frac{5}{7} + \left(-1\frac{1}{3}\right)$

   $2\frac{15}{21} - 1\frac{7}{21} = 1\frac{8}{21}$

5. $-\frac{3}{4} + \frac{5}{8}$

   $-\frac{6}{8} + \frac{5}{8} = -\frac{1}{8}$

6. $-7.38 - (-6.2)$

   -1.18

7. $-\frac{3}{4} - \frac{5}{8}$

   $-\frac{6}{8} - \frac{5}{8} = -\frac{11}{8}$

8. $-2\frac{5}{6} + 1\frac{3}{8}$

   $-2\frac{20}{24} + 1\frac{9}{24} = -1\frac{11}{24}$

9. $-\frac{7}{12} - \frac{5}{6}$

   $-\frac{7}{12} - \frac{10}{12} = -\frac{17}{12}$

10. $-37.27 + (-13.2)$

    -50.47

11. $-0.8 - (-0.6)$

-0.2

12. $2\frac{3}{7} + -1\frac{3}{4}$

$2\frac{12}{28} - 1\frac{21}{28} = \frac{19}{28}$

13. $0.67 + (-0.33)$

0.34

14. $-42.65 - (-16.3)$

-26.35

15. $-7300 + 2100$

-5200

16. $-3\frac{5}{8} - \left(-2\frac{1}{3}\right)$

$-3\frac{15}{24} + 2\frac{8}{24} = -1\frac{7}{24}$

17. $-4.7 + 3.16$

-1.54

18. $26.9 - (-3.1)$

30

19. $-325 + (-775)$

-1100

20. $-2\frac{1}{5} - 1\frac{3}{10}$

$-2\frac{2}{10} - 1\frac{3}{10} = -3\frac{5}{10}$

$= -3\frac{1}{2}$

Be prepared to share your solutions and methods.

Follow Up

Assignment

Use the Assignment for Lesson 4.5 in the Student Assignments book. See the Teacher's Resources and Assessments book for answers.

Skills Practice

Refer to the Skills Practice worksheet for Lesson 4.5 in the Student Assignments book for additional resources. See the Teacher's Resources and Assessments book for answers.

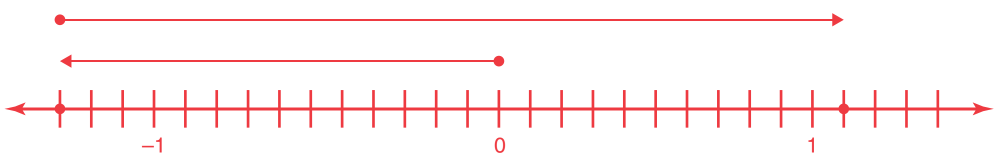
Assessment

See the Assessments provided in the Teacher's Resources and Assessments book for Chapter 4.

Check for Students' Understanding

Consider the problem $-1.3 - (-2.4)$

1. Use a number line to solve this problem.



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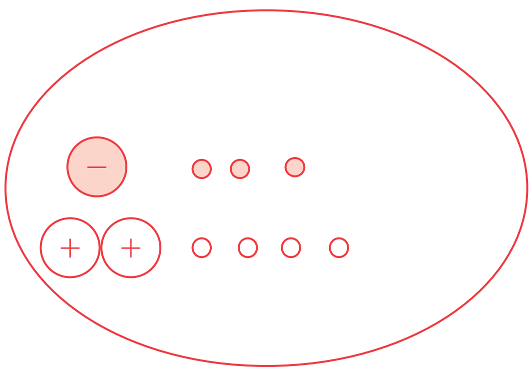
2. Use a two-counter model to solve this problem.



3. Use an algorithm or rule to solve this problem.

$-1.3 - (-2.4) = -1.3 + 2.4 = 1.1$

Chapter 4 Summary

Key Terms

- additive inverses (4.3)
- zero pair (4.4)

4.1 Writing Number Sentences to Represent the Sum of Positive and Negative Integers

Integers are useful for representing some sort of progress from a starting quantity or position. Sequential events can often be modeled by a number sentence involving both positive and negative integers.

Example

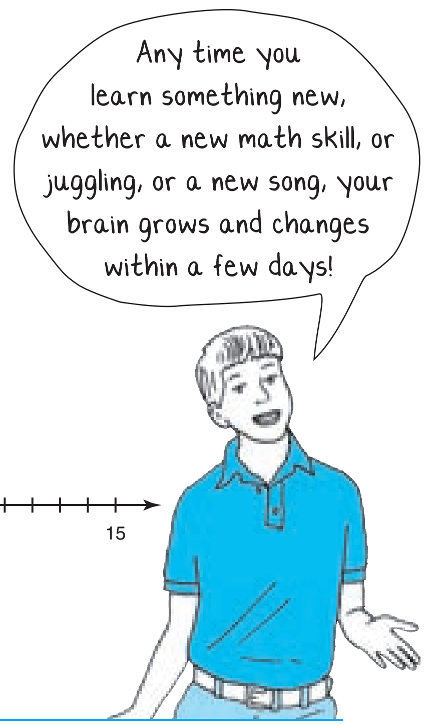
During a model boat race, a boat is in the lead by two boat lengths at the halfway point of the race. However, during the second half of the race, the boat loses five boat lengths to the eventual winner. The boat's progress in relation to the boat race winner is shown through the additional sentence.

$(+2) + (-5) = -3$

4

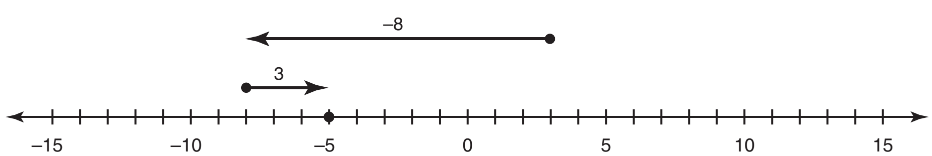
4.2 Modeling Integer Addition on a Number Line

A number line can be used to model integer addition. When adding a positive integer, move to the right on the number line. When adding a negative integer, move to the left on the number line.



Example

$-8 + 3$



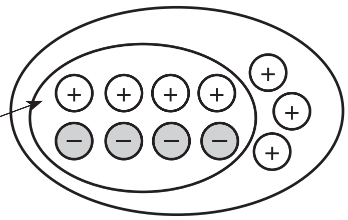
$-8 + 3 = -5$

4.3 Modeling Integer Addition Using Two-Color Counters

Let a red counter represent -1 and a yellow counter represent $+1$. Each pair of positive and negative counters has a value of zero.

Example

A model representing $7 + (-4)$ using two-color counters is shown. The zero pairs are circled showing the sum.



$7 + (-4) = 3$

4

4.3 Adding Integers

When adding two integers with the same sign, add the integers and keep the sign. When adding integers with opposite signs, subtract the integers and keep the sign of the integer with the greater absolute value.

Example

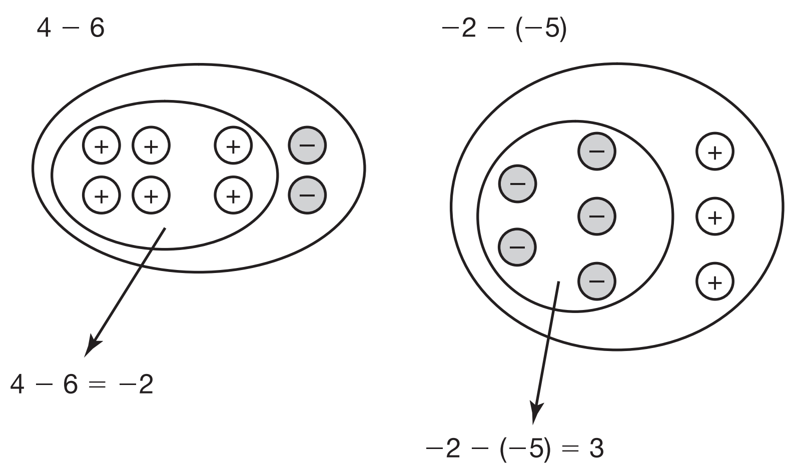
$-9 + (-12)$	$7 + (-15)$
$= -(9 + 12)$	$= -8$
$= -21$	

4.4 Modeling Integer Subtraction Using Two-Color Counters

Subtraction can be modeled by "taking away" objects of a set. Positive and negative counters can be used to represent this "take away" model. Because a pair of positive and negative counters has a value of zero, as many zero pairs as are needed can be added without changing the value.

Example

Two-color counters can be used to model subtraction. Begin by adding the number of counters to represent the first term, and then add enough zero pairs to be able to subtract the second term.

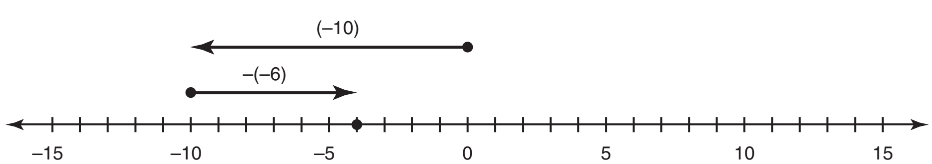


4.4 Modeling Integer Subtraction on a Number Line

A number line can be used to model integer subtraction. Subtraction means to move in the opposite direction on the number line. When subtracting a positive integer, move to the left on the number line. When subtracting a negative integer, move to the right on the number line.

Example

$-10 - (-6)$



$-10 - (-6) = -4$

4.4 Subtracting Integers

Because subtraction is the inverse of addition, it is the same as adding the opposite number.

Examples

$-7 - 19 = -7 + (-19)$
$= -26$

$12 - 21 = 12 + (-21)$
$= -9$

4.5 Adding Rational Numbers

When adding positive and negative rational numbers, follow the same rules as when adding integers. When adding rational numbers with the same sign, add the numbers and keep the sign. When the rational numbers have different signs, subtract the numbers and keep the sign of the number with the greater absolute value.

Examples

$-8.54 + (-3.4)$
$= -(8.54 + 3.4)$
$= -11.94$

$5\frac{1}{2} + (-10\frac{3}{4})$
$= 10\frac{3}{4} - 5\frac{2}{4}$
$= -5\frac{1}{4}$

4

4.5 Subtracting Rational Numbers

When subtracting positive and negative rational numbers, follow the same rules as when subtracting integers. Because subtraction is the inverse of addition, it is the same as adding the opposite number.

Examples

$-7\frac{1}{4} - (-10\frac{5}{8})$
$= -7\frac{2}{8} + (+10\frac{5}{8})$
$= 3\frac{3}{8}$

$-8.5 - 3.4$
$= -8.5 + (-3.4)$
$= -11.9$