6 NUMERICAL AND ALGEBRAIC EXPRESSIONS AND EQUATIONS

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Chapter 6 Overview

This chapter focuses on the use of properties to interpret, simplify, add, subtract, and factor linear expressions.

Lessons		CCSS	Pacing	Highlights	Models	Worked Examples	Peer Analysis	Talk the Talk	Technology
6.1	Evaluating Algebraic Expressions	6.EE.6 7.NS.3	1	This lesson explores the use of algebraic expressions as efficient representations for repeating patterns. Questions ask students to use tables to organize values when evaluating algebraic expressions.	X			X	
6.2	Simplifying Expressions Using Distributive Properties	7.NS.3 7.EE.1 7.EE.2	1	This lesson reviews all aspects of the distributive properties. Questions ask students to analyze models and explore various strategies for simplifying expressions to gain a deep understanding of how the properties work.	X		X		
6.3	Factoring Algebraic Expressions	7.EE.1 7.EE.2	1	This lesson continues the development of distributive properties to simplify and factor expressions.		X			
6.4	Verifying That Expressions Are Equivalent	7.EE.1 7.EE.2	1	This lesson explores different methods, including the use of graphing calculators, to determine if expressions are equivalent.			X		X
6.5	Simplifying Algebraic Expressions Using Operations and Their Properties	7.EE.1 7.EE.2	1	This lesson requires the use of operations and properties to justify the steps of the simplification process to determine if expressions are equivalent.		X			

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Skills Practice Correlation for Chapter 6

Lessons		Problem Set	Objective(s)
6.1	Evaluating Algebraic Expressions		Vocabulary
		1 – 6	Define variables and write algebraic expressions
		7 – 14	Evaluate algebraic expressions
		15 – 20	Complete tables of expressions
		21 – 26	Evaluate algebraic expressions for given quantities
6.2	Simplifying Expressions Using Distributive Properties		Vocabulary
		1 – 10	Draw models and calculate or simplify expressions
		11 – 20	Use the Distributive Property to rewrite expressions
		21 – 26	Evaluate expressions for given values
6.3	Factoring Algebraic Expressions		Vocabulary
		1 – 10	Rewrite expressions by factoring out the GCF
		11 – 20	Simplify expressions by combining like terms
		21 – 26	Evaluate expressions for given values by factoring
		27 – 32	Evaluate expressions for given values by combining like terms
6.4	Verifying That Expressions Are Equivalent	1 – 6	Determine whether expressions are equivalent by evaluating for given values
		7 – 12	Determine whether expressions are equivalent by simplifying
		13 – 18	Determine whether expressions are equivalent by graphing
6.5	Simplifying Algebraic Expressions Using Operations and Their Properties	1 – 8	Complete justification tables to determine if expressions are equivalent
		9 – 14	Give justification of steps to determine if expressions are equivalent by simplifying

6

6.1 What's It Really Saying?

Evaluating Algebraic Expressions

Learning Goal

In this lesson, you will:

- Evaluate algebraic expressions.

Key Terms

- variable
- algebraic expression
- evaluate an algebraic expression

Essential Ideas

- An algebraic expression is a mathematical phrase involving at least one variable, and it can contain numbers and operational symbols.
- An algebraic expression is often used to represent situations in which the same mathematical process is performed over and over again.
- To evaluate an expression, you replace each variable in the expression with numbers and then perform all possible mathematical operations.
- When you substitute known measures into a formula and simplify it to determine a new measure, you are evaluating the formula.

Common Core State Standards for Mathematics

6.EE Expressions and Equations

Reason about and solve one-variable equations and inequalities.

6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

7.NS The Number System

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

3. Solve real-world and mathematical problems involving the four operations with rational numbers.

Overview

Students are introduced to algebraic expressions and variables as an efficient method to represent situations where the same mathematical process is repeated over and over again. In the first two problems, students write and use algebraic expressions to represent given contexts. Students are introduced to the terminology “evaluating an algebraic expression” and will practice this skill in the remaining two problems. Students complete tables that record the results when evaluating the same expression with multiple values.

6

Warm Up

1. Simplify each numeric expression.

   a. $8 + 9 \cdot 2$

   26

   b. $8 + 9 \cdot 3$

   35

   c. $8 + 9 \cdot 4$

   44

   d. $8 + 9 \cdot 5$

   53

   e. $8 + 9 \cdot 6$

   62

   f. $8 + 9 \cdot 7$

   71

   g. $8 + 9 \cdot 8$

   80

   h. $8 + 9 \cdot 9$

   89

2. Explain the rule you used to simplify these problems?

   I use the order of operations rules and multiplied before I added.

3. What was the same in each problem?

   Every problem has 9 times a number, and then 8 added to it.

4. What was different in each problem?

   The number 9 was multiplied by a different value every time.

5. What patterns did you notice in your solutions?

   The solutions increased by 9 every time. Except for the last solution, the tens-digit increased by one every time and the ones-digit decreased by one every time.

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6.1 WHAT'S IT REALLY SAYING?

Evaluating Algebraic Expressions

Learning Goal

In this lesson, you will:

- Evaluate algebraic expressions.

Key Terms

- variable
- algebraic expression
- evaluate an algebraic expression

Do you have all your ducks in a row? That's just a drop in the bucket! That's a piece of cake!

What do each of these statements have in common? Well, they are all idioms. Idioms are expressions that have meanings which are completely different from their literal meanings. For example, the "ducks in a row" idiom refers to asking if someone is organized and ready to start a task. A person who uses this idiom is not literally asking if you have ducks that are all lined up.

For people just learning a language, idioms can be very challenging to understand. Usually if someone struggles with an idiom's meaning, a person will say "that's just an expression," and explain its meaning in a different way. Can you think of other idioms? What does your idiom mean?

Problem 1

A context is given and students complete a table using repeated multiplication to determine a total cost. The repeated calculations provide the background to introduce algebraic expressions and variables as an efficient method to represent situations where the same mathematical process is repeated over and over again. Students will write and use the algebraic expression for the given context.

Grouping

Have students complete Questions 1 through 3 with a partner. Then share the responses as a class.

Share Phase, Questions 1 through 3

- What is the advantage of having a Game Day special for the customers?
- What is the advantage of having a Game Day special for the sellers?
- Why was it suggested that you multiply to determine the costs?
- Could you have gotten those same results by using another mathematical operation? Explain.
- How is the term variable related to its root word "vary"?
- Which of these are algebraic expressions: $\frac{3}{4} + 8$, $\frac{3}{4}x + 8$, x, $xy - z$ and 9?

Problem 1 Game Day Special

You volunteer to help out in the concession stand at your middle-school football game. You must create a poster to display the Game Day Special: a hot dog, a bag of chips, and a drink for $3.75.

1. Complete the poster by multiplying the number of specials by the cost of the special.

Game Day Special 1 hot dog, 1 bag of chips, and a drink for $3.75	
Number of Specials	Cost
1	$3.75
2	$7.50
3	$11.25
4	$15.00
5	$18.75
6	$22.50

In algebra, a **variable** is a letter or symbol that is used to represent a quantity. An **algebraic expression** is a mathematical phrase that has at least one variable, and it can contain numbers and operation symbols.

Whenever you perform the same mathematical process over and over again, an algebraic expression is often used to represent the situation.

2. What algebraic expression did you use to represent the total cost on your poster? Let s represent the number of Game Day Specials.

   The algebraic expression I used was $3.75s$.

3. The cheerleading coach wants to purchase a Game Day Special for every student on the squad. Use your algebraic expression to calculate the total cost of purchasing 18 Game Day Specials.

   $3.75(18) = 67.50$

   The total cost of 18 Game Day Specials is $67.50.

- How did your algebraic expression help you solve for the total cost of purchasing 18 Game Day Specials?
- Is there another way to write the algebraic expression?

Note

Encourage students to use parentheses when substituting values in algebraic expressions. It makes the substitution more visible.

Problem 2

Students write an algebraic expression including both multiplication and addition to represent a context. They will use the algebraic expression to calculate the results for multiple values of the variable; this time the results are not represented in a table.

Grouping

Have students complete Questions 1 and 2 with a partner. Then share the responses as a class.

Share Phase, Questions 1 and 2

- Is there another way to write the algebraic expression?
- How is this algebraic expression different from the one in Problem 1?
- How did your algebraic expression help you solve for the cost of the party for each number of guests?
- What makes more sense in the context of this situation, the expression $(105 + 40)g$ or the expression $(105 + 40g)$? Explain.
- What makes more sense in the context of this situation, the expression $(105 + 40g)$ or the expression $(40g + 105)$? Explain.

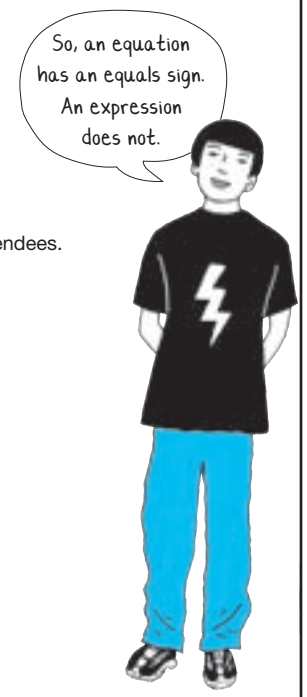
Problem 2 Planning a Graduation Party

Your aunt is planning to host your cousin's high school graduation party at Lattanzi's Restaurant and Reception Hall. Lattanzi's has a flyer that describes the Deluxe Graduation Reception.

> Deluxe Graduation Reception
>
> Includes:
> One salad (chef or Caesar)
> One entree (chicken, beef, or seafood)
> Two side dishes
> One dessert
>
> Fee:
> $105 for the reception hall plus $40 per guest

1. Write an algebraic expression to determine the cost of the graduation party. Let g represent the number of guests attending the party.

   $105 + 40g$



2. Determine the cost of the party for each number of attendees. Show your work.

   a. 8 guests attend

   The party would cost $425 if 8 guests attended.
   $105 + 40(8) = 425$

   b. 10 guests attend

   The party would cost $505 if 10 guests attended.
   $105 + 40(10) = 505$

   c. 12 guests attend

   The party would cost $585 if 12 guests attended.
   $105 + 40(12) = 585$

Note

The expression $105 + 40g$ will probably make more sense to students than $40g + 105$ because it follows the context chronologically; 105 is the initial fee and then you pay for the guests. Follow what makes sense to students, it is not necessary to guide students to $y = mx + b$ form at this time.

Problem 3

Students are introduced formally to the terminology "evaluate an algebraic expression". They will evaluate algebraic expressions represented both with and without tables.

Grouping

Have students complete Question 1 with a partner. Then share the responses as a class.

Share Phase, Question 1

- What does it mean to evaluate an expression?
- How is the term evaluate related to its root word "value"?
- What integer rule did you use to evaluate this expression?
- How is this question similar to those in the previous problems?
- How is this question different from those in previous problems?

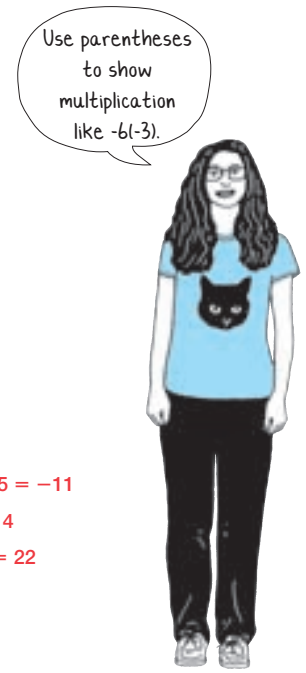
Problem 3 Evaluating Expressions

In Problems 1 and 2, you worked with two expressions, $3.75s$ and $(105 + 40g)$. You evaluated those expressions for different values of the variable. To **evaluate an algebraic expression**, you replace each variable in the expression with a number or numerical expression and then perform all possible mathematical operations.

1. Evaluate each algebraic expression.

a. $x - 7$

- for $x = -8$ $\quad x - 7 = -8 - 7 = -15$
- for $x = -11$ $\quad x - 7 = -11 - 7 = -18$
- for $x = 16$ $\quad x - 7 = 16 - 7 = 9$



b. $-6y$

- for $y = -3$ $\quad -6y = -6(-3) = 18$
- for $y = 0$ $\quad -6y = -6(0) = 0$
- for $y = 7$ $\quad -6y = -6(7) = -42$

c. $3b - 5$

- for $b = -2$ $\quad 3b - 5 = 3(-2) - 5 = -6 - 5 = -11$
- for $b = 3$ $\quad 3b - 5 = 3(3) - 5 = 9 - 5 = 4$
- for $b = 9$ $\quad 3b - 5 = 3(9) - 5 = 27 - 5 = 22$

d. $-1.6 + 5.3n$

- for $n = -5$ $\quad -1.6 + 5.3n = -1.6 + 5.3(-5) = -1.6 + (-26.5) = -28.1$
- for $n = 0$ $\quad -1.6 + 5.3n = -1.6 + 5.3(0) = -1.6 + 0 = -1.6$
- for $n = 4$ $\quad -1.6 + 5.3n = -1.6 + 5.3(4) = -1.6 + 21.2 = 19.6$

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Grouping

Have students complete Question 2 with a partner. Then share the responses as a class.

Share Phase, Question 2

- What mathematical operations are involved in this question?
- Where is the substitution represented in your work?
- What integer rules did you use to evaluate this expression?
- What makes a table more convenient than a series of questions?

Sometimes, it is more convenient to use a table to record the results when evaluating the same expression with multiple values.

2. Complete each table.

a.

h	$-2h - 7$
2	−11
−1	−5
8	−23
−7	7

b.

a	−12	−10	−4	0
$\frac{a}{4} + 6$	3	3.5	5	6

c.

x	$x^2 - 5$
1	−4
3	4
6	31
−2	−1

d.

y	−5	−1	0	15
$-\frac{1}{5}y + 3\frac{2}{5}$	$4\frac{2}{5}$	$3\frac{3}{5}$	$3\frac{2}{5}$	$\frac{2}{5}$

Problem 4

Students use integers, decimals, and mixed numbers to evaluate algebraic expressions.

Grouping

Have students complete Questions 1 through 3 with a partner. Then share the responses as a class.

Share Phase, Question 1

- When evaluating the expression, which value was easiest to work with? Why?
- When evaluating the expression, which value was the most difficult to work with? Why?

Problem 4 Evaluating Algebraic Expressions Using Given Values

1. Evaluate each algebraic expression for $x = 2, -3, 0.5$, and $-2\frac{1}{3}$.

a. $-3x$

$-3(2) = -6$

$-3(-3) = 9$

$-3(0.5) = -1.5$

$-3\left(-2\frac{1}{3}\right) = 7$

b. $5x + 10$

$5(2) + 10 = 20$

$5(-3) + 10 = -5$

$5(0.5) + 10 = 12.5$

$5\left(-2\frac{1}{3}\right) + 10 = -11\frac{2}{3} + 10 = -1\frac{2}{3}$

c. $6 - 3x$

$6 - 3(2) = 0$

$6 - 3(-3) = 15$

$6 - 3(0.5) = 4.5$

$6 - 3\left(-2\frac{1}{3}\right) = 6 + 7 = 13$

d. $8x + 75$

$8(2) + 75 = 91$

$8(-3) + 75 = 51$

$8(0.5) + 75 = 79$

$8\left(-2\frac{1}{3}\right) + 75 = 56\frac{1}{3}$

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Share Phase, Question 2

- When evaluating the expression, which value was easiest to work with? Why?
- When evaluating the expression, which value was the most difficult to work with? Why?

2. Evaluate each algebraic expression for $x = -7$, 5, 1.5, and $-1\frac{1}{6}$.

a. $5x$

$5(-7) = -35$

$5(5) = 25$

$5(1.5) = 7.5$

$5\left(-1\frac{1}{6}\right) = -5\frac{5}{6}$

b. $2x + 3x$

$2(-7) + 3(-7) = -35$

$2(5) + 3(5) = 25$

$2(1.5) + 3(1.5) = 7.5$

$2\left(-1\frac{1}{6}\right) + 3\left(-1\frac{1}{6}\right) = -5\frac{5}{6}$

c. $8x - 3x$

$8(-7) - 3(-7) = -35$

$8(5) - 3(5) = 25$

$8(1.5) - 3(1.5) = 7.5$

$8\left(-1\frac{1}{6}\right) - 3\left(-1\frac{1}{6}\right) = -5\frac{5}{6}$

Share Phase, Question 3

- When evaluating the expression, which value was easiest to work with? Why?
- When evaluating the expression, which value was the most difficult to work with? Why?

Talk the Talk

Students describe a strategy for evaluating an algebraic expression.

Grouping

Have students complete Questions 1 and 2 independently. Then share the responses as a class.

3. Evaluate each algebraic expression for $x = 23.76$ and $-21\frac{5}{6}$.

a. $2.67x - 31.85$

$2.67(23.76) - 31.85 = 31.5892$

$2.67\left(-21\frac{5}{6}\right) - 31.85 = -90.145$

b. $11\frac{3}{4}x + 56\frac{3}{8}$

$11\frac{3}{4}(23.76) + 56\frac{3}{8} = 335.555$

$11\frac{3}{4}\left(-21\frac{5}{6}\right) + 56\frac{3}{8}$

$\frac{47}{4}\left(-\frac{131}{6}\right) + 56\frac{3}{8} =$

$-\frac{6157}{24} + \frac{1353}{24} = -200\frac{1}{6}$

Talk the Talk

1. Describe your basic strategy for evaluating any algebraic expression.

I substitute a value for the variable and then follow the order of operation rules.

2. How are tables helpful when evaluating expressions?

Tables help me organize the values when I evaluate an expression.

Be prepared to share your solutions and methods.

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Follow Up

Assignment

Use the Assignment for Lesson 6.1 in the Student Assignments book. See the Teacher's Resources and Assessments book for answers.

Skills Practice

Refer to the Skills Practice worksheet for Lesson 6.1 in the Student Assignments book for additional resources. See the Teacher's Resources and Assessments book for answers.

Assessment

See the Assessments provided in the Teacher's Resources and Assessments book for Chapter 6.

Check for Students' Understanding

Use a variable to write an algebraic expression to represent each series of numerical expressions. After each algebraic expression, list the values for the variable.

1. $5 \cdot 3 + 7$
$5 \cdot 4 + 7$
$5 \cdot 5 + 7$
$5 \cdot 6 + 7$

$5 \cdot x + 7$
$x = 3, 4, 5$ and 6

2. $2 \cdot 8 + 7$
$6 \cdot 8 + 7$
$9 \cdot 8 + 7$
$8 \cdot 8 + 7$

$y \cdot 8 + 7$
$y = 2, 6, 9$ and 8

3. $3 \cdot 8 + 9$
$3 \cdot 8 + 11$
$3 \cdot 8 + 6$
$3 \cdot 8 + 25$

$3 \cdot 8 + z$
$z = 9, 11, 6$ and 25

4. $11(5 + 9)$
$11(9 + 8)$
$(12 + 9)11$
$11 \cdot (9 + 9)$

$11(x + 9)$
$x = 5, 8, 12$, and 9

6

6.2 Express Math

Simplifying Expressions Using Distributive Properties

Learning Goals

In this lesson, you will:

- Write and use the distributive properties.
- Use distributive properties to simplify expressions.

Key Terms

- Distributive Property of Multiplication over Addition
- Distributive Property of Multiplication over Subtraction
- Distributive Property of Division over Addition
- Distributive Property of Division over Subtraction

Essential Ideas

- The Distributive Property provides ways to write numerical and algebraic expressions in equivalent forms.
- When the Distributive Property is applied to numerical expressions only, it is a helpful tool in computing math mentally.
- The area of a rectangle model is useful in demonstrating the Distributive Property.
- There are four versions of the distributive property:
  - Distributive Property of Multiplication over Addition: If a, b and c are any real numbers, then $a \cdot (b + c) = a \cdot b + a \cdot c$.
  - Distributive Property of Multiplication over Subtraction: If a, b and c are any real numbers, then $a \cdot (b - c) = a \cdot b - a \cdot c$.
  - Distributive Property of Division over Addition: If a, b, and c are any real numbers and $c \neq 0$, then $\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$.
  - Distributive Property of Division over Subtraction: If a, b and c are any real numbers and $c \neq 0$, then $\frac{a - b}{c} = \frac{a}{c} - \frac{b}{c}$.

Common Core State Standards for Mathematics

7.NS The Number System

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

3. Solve real-world and mathematical problems involving the four operations with rational numbers.

7.EE Expressions and Equations

Use properties of operations to generate equivalent expressions.

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

Overview

The Distributive Property is introduced. Students explore the distributive properties by calculating the product of two numbers. Two methods are given and discussed. One model involves computing the multiplication problem using mental math and the other model involves using the areas of rectangles.

The Distributive Property of Multiplication and Division over Addition and Subtraction are introduced. Students simplify algebraic expressions using both the area model and symbolic representations. They will then reverse the process of the Distributive Property of Multiplication over Addition and Subtraction to factor numerical and algebraic expressions.

Students are guided to use the Distributive Property of Multiplication over Addition and Subtraction to combine like terms. They will develop the understanding that the coefficients of like terms are added, while the variable remains the same.

Warm Up

1. Rewrite each fraction in reduced form.

a. $\frac{9}{24}$ $\frac{3}{8}$

b. $\frac{55}{88}$ $\frac{5}{8}$

c. $\frac{21}{28}$ $\frac{3}{4}$

d. $\frac{13}{26}$ $\frac{1}{2}$

e. What is the mathematical term of the divisor used to reduce the fractions?

The value I used to divide by to reduce the fractions is called the greatest common factor or GCF.

2. The value 642 can be written as a sum of several values.

- $642 = 321 + 321$
- $642 = 600 + 40 + 2$
- $642 = 200 + 221 + 221$

Which representation do you think is easiest to work with? Why?

642 = 600 + 40 + 2 seems to be the simplest representation because it uses the place values.

3. Rewrite each value as the sum of several values using the simplest representation.

a. 937 $900 + 30 + 7$

b. 10,500 $10{,}000 + 500$

c. 702 $700 + 2$

d. 8040 $8000 + 40$

4. The value 295 can be written as the sum of 200 + 90 + 5. A simpler representation may be written using differences because 295 is close to 300. For example, $295 = 300 - 5$.

Why is the representation using differences considered simpler?

The difference takes advantage of the fact that the number is close to a rounded value, and it only contains two terms.

5. Rewrite each value as the difference of two values using the simplest representation.

a. 398 $400 - 2$

b. 79 $80 - 1$

c. 995 $1000 - 5$

d. 9992 $10{,}000 - 8$

6

6.2 EXPRESS MATH

Simplifying Expressions Using Distributive Properties

Learning Goals

In this lesson, you will:

- Write and use the distributive properties.
- Use distributive properties to simplify expressions.

Key Terms

- Distributive Property of Multiplication over Addition
- Distributive Property of Multiplication over Subtraction
- Distributive Property of Division over Addition
- Distributive Property of Division over Subtraction

It once started out with camping out the night before the sale. Then, it evolved to handing out wrist bands to prevent camping out. Now, it's all about the Internet. What do these three activities have in common?

For concerts, movie premieres, and highly-anticipated sporting events, the distribution and sale of tickets have changed with computer technology. Generally, hopeful ticket buyers log into a Web site and hope to get a chance to buy tickets. What are other ways to distribute tickets? What are other things that routinely get distributed to people?

Problem 1

This problem addresses all aspects of the distributive property. Students explore the distributive property by calculating the product of two numbers in two ways. They will analyze two models, a method for computing mental math and the area of rectangles diagram, and they connect and use both models. The Distributive Properties of Multiplication over Addition, Multiplication over Subtraction, Division over Addition, and Division over Subtraction are introduced. Students will simplify algebraic expressions using both the area model and symbolic representations. They then reverse the process of the Distributive Property of Multiplication over Addition and Subtraction, and factor numerical and algebraic expressions.

Grouping

Have students complete Questions 1 through 3 with a partner. Then share the responses as a class.

6

Share Phase, Questions 1 and 2

- Are you able to calculate the product using mental math? If so, explain your process.
- Why is the area model appropriate for this calculation?

Problem 1 Fastest Math in the Wild West

Dominique and Sarah are checking each other's math homework. Sarah tells Dominique that she has a quick and easy way to multiply a one-digit number by any other number in her head. Dominique decides to test Sarah by asking her to multiply 7 by 230.

Sarah is able to correctly multiply 7 by 230 in her head. She explains her method to Dominique.

Sarah

230 × 7

First, break 230 into the sum of 200 and 30. Then, multiply 7 × 200 to get a product of 1400 and 7 × 30 to get a product of 210. Finally, add 1400 and 210 together to get 1610.

1. Calculate the product 7 × 230.

   230
   × 7
   1610

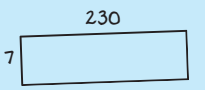
2. Write an expression that shows the mathematical steps Sarah performed to calculate the product.

   7(200) + 7(30)

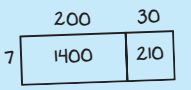
Dominique makes the connection between Sarah's quick mental calculation and calculating the area of a rectangle. She draws the models shown to represent Sarah's thinking in a different way.

Dominique

Calculating 230 × 7 is the same as determining the area of a rectangle by multiplying the length by the width.



But I can also divide the rectangle into two smaller rectangles and calculate the area of each rectangle. I can then add the two areas to get the total.



1400 + 210 = 1610

- Why was 230 represented as 200 + 30 rather than another sum?
- What do the numbers in the boxes represent?
- Explain the area model in your own words.
- How are the area model and the mental math model similar?

Share Phase, Questions 3

- Why did you choose the particular sum that you did to represent the two- or three-digit number?
- Where are the values from your use of Sarah's method represented in your area model?
- Explain your use of parentheses when you used Sarah's method.

Grouping

Ask a student to read the information following Question 3 aloud. Discuss the information as a class.

Discuss Phase, Definitions

- Why do you think it is called the Distributive Property of Multiplication over Addition?
- Use one of the items from Question 3 to demonstrate the substitution of values for *a*, *b* and *c* in the formal definition.
- Use 4(499) to demonstrate using the Distributive Property of Multiplication over Subtraction.
- Use $\frac{535}{5}$ to demonstrate using the Distributive Property of Division over Addition.

3. First, use Dominique's method and sketch a model for each. Then, write an expression that shows Sarah's method and calculate.

a. 9(48)

	40	8
9	360	72

9(40 + 8)
= 9(40) + 9(8)
= 360 + 72 = 432

b. 6(73)

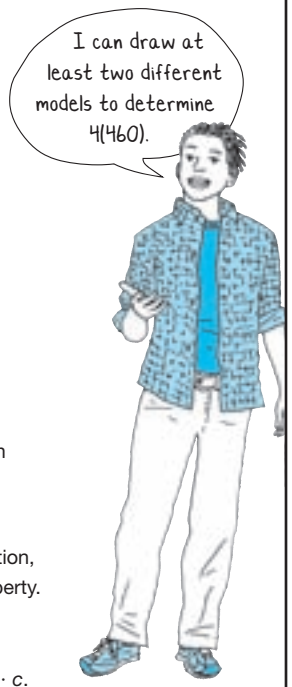
	70	3
6	420	18

6(70 + 3)
= 6(70) + 6(3)
= 420 + 18 = 438

c. 4(460)

	400	60
4	1600	240

4(400 + 60)
= 4(400) + 4(60)
= 1600 + 240 = 1840



Sarah's and Dominique's methods are both examples of the **Distributive Property of Multiplication over Addition,** which states that if *a*, *b*, and *c* are any real numbers, then $a \cdot (b + c) = a \cdot b + a \cdot c$.

Including the Distributive Property of Multiplication over Addition, there are a total of *four* different forms of the Distributive Property. Another Distributive Property is the **Distributive Property of Multiplication over Subtraction,** which states that if *a*, *b*, and *c* are any real numbers, then $a \cdot (b - c) = a \cdot b - a \cdot c$.

- Use $\frac{595}{5}$ to demonstrate using the Distributive Property of Division over Subtraction.
- Explain when it may more appropriate to use subtraction rather than addition.

Grouping

Have students complete Questions 4 and 5 with a partner. Then share the responses as a class.

Share Phase, Questions 4 and 5

- How are these problems in Question 4 different from the previous problems?
- Why can't you simplify your algebraic expressions in Question 4 by adding the areas?
- How did you determine the signs of each term?
- Is the process any different if there are more than two terms in the parentheses?
- Explain the distributive process for the division problems.

Misconceptions

- Be sure that students do not over generalize the Distributive Property of Division over Addition or Subtraction. The numerator can be split as a sum for ease in calculations; but the denominator cannot be split as a sum.

  For example: $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

  For example: $\frac{c}{a+b} \neq \frac{c}{a} + \frac{c}{b}$

The Distributive Property also holds true for division over addition and division over subtraction as well.

The **Distributive Property of Division over Addition** states that if a, b, and c are real numbers and $c \neq 0$, then $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$.

The **Distributive Property of Division over Subtraction** states that if a, b, and c are real numbers and $c \neq 0$, then $\frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$.

4. Draw a model for each expression, and then simplify.

a. $6(x + 9)$

	x	9
6	$6x$	54

$6x + 54$

b. $7(2b - 5)$

	$2b$	-5
7	$14b$	-35

$14b - 35$

c. $-2(4a + 1)$

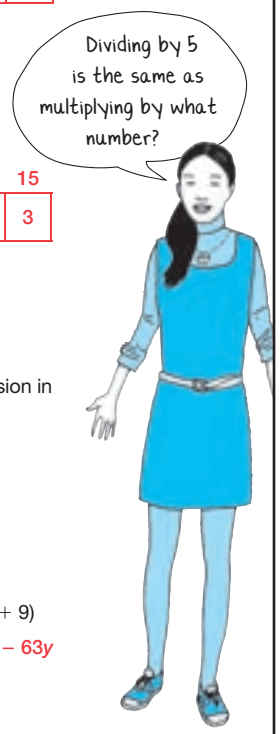
	$4a$	1
-2	$-8a$	-2

$-8a - 2$

d. $\frac{x + 15}{5}$

	x	15
$\frac{1}{5}$	$\frac{1}{5}x$	3

$\frac{x}{5} + 3$



5. Use one of the Distributive Properties to rewrite each expression in an equivalent form.

a. $3y(4y + 2)$

$12y^2 + 6y$

b. $12(x + 3)$

$12x + 36$

c. $-4a(3b - 5)$

$-12ab + 20a$

d. $-7y(2y - 3x + 9)$

$-14y^2 + 21xy - 63y$

e. $\frac{6m + 12}{-2}$

$-3m - 6$

f. $\frac{22 - 4x}{2}$

$11 - 2x$

- A common error is that students will distribute to the first term in the parentheses, but forget to distribute to the second term. Use of the area model helps eliminate this error by having students fill in a rectangle for each term.

Problem 2

Students simplify five expressions using distributive properties. They will then evaluate two expressions using distributive properties.

Grouping

Have students complete Questions 1 and 2 with a partner. Then share the responses as a class.

Share Phase, Question 1

- How can a distributive property be used to simplify this expression?
- What is the first step?
- What is the second step?
- Are there any like terms?
- What can be combined?

Problem 2 Simplifying and Evaluating

1. Simplify each expression. Show your work.

a. $-6(3x + (-4y))$

$$-6(3x + (-4y)) = (-6)(3x) + (-6)(-4y)$$
$$= -18x + 24y$$

b. $-4(-3x - 8) - 34$

$$-4(-3x - 8) - 34 = (-4)(-3x) + (-4)(-8) + (-34)$$
$$= 12x + 32 + (-34)$$
$$= 12x + (-2)$$

c. $\dfrac{-7.2 - 6.4x}{-0.8}$

$$\frac{-7.2 + 6.4x}{-0.8} = \frac{-7.2}{-0.8} + \frac{6.4x}{-0.8}$$
$$= 9 + (-8x)$$

d. $\left(-2\frac{1}{2}\right)\left(3\frac{1}{4}\right) + \left(-2\frac{1}{2}\right)\left(-2\frac{1}{4}\right)$

$$\left(-2\tfrac{1}{2}\right)\left(3\tfrac{1}{4}\right) + \left(-2\tfrac{1}{2}\right)\left(-2\tfrac{1}{4}\right) = \left(-2\tfrac{1}{2}\right)\left(3\tfrac{1}{4} + \left(-2\tfrac{1}{4}\right)\right)$$
$$= \left(-2\tfrac{1}{2}\right)(1)$$
$$= -2\tfrac{1}{2}$$

e. $\dfrac{\left(-7\frac{1}{2}\right) + 5y}{2\frac{1}{2}}$

$$\frac{\left(-7\frac{1}{2}\right) + 5y}{2\frac{1}{2}} = \frac{-7\frac{1}{2}}{2\frac{1}{2}} + \frac{5y}{2\frac{1}{2}}$$
$$= -3 + 2y$$

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Share Phase, Question 2

- Did you use any of the distributive properties to simplify this expression? Which one(s)?
- Is the answer in the simplest form? How do you know?

2. Evaluate each expression for the given value. Then, use properties to simplify the original expression. Finally, evaluate the simplified expression.

a. $2x(-3x + 7)$ for $x = -1\frac{2}{3}$

$$\begin{aligned} 2x(-3x+7) &= 2\left(-1\tfrac{2}{3}\right)\left(-3\left(-1\tfrac{2}{3}\right)+7\right) \\ &= \frac{2}{1}\left(-\frac{5}{3}\right)\left(-\frac{\cancel{3}^{1}}{1}\left(-\frac{5}{\cancel{3}_{1}}\right)+7\right) \\ &= -\frac{10}{3}(5+7) \\ &= -\left(\frac{10}{\cancel{3}_{1}}\right)\left(\frac{\cancel{12}^{4}}{1}\right) \\ &= -40 \end{aligned}$$

$$\begin{aligned} 2x(-3x+7) &= 6x^2 + 14x \\ &= -6\left(-1\tfrac{2}{3}\right)^2 + 14\left(-1\tfrac{2}{3}\right) \\ &= \left(-\frac{\cancel{6}^{2}}{1}\right)\left(-\frac{5}{\cancel{3}_{1}}\right)\left(-\frac{5}{3}\right) + \frac{14}{1}\left(-\frac{5}{3}\right) \\ &= -\frac{50}{3} + \left(-\frac{70}{3}\right) \\ &= -\frac{120}{3} \\ &= -40 \end{aligned}$$

b. $\frac{4.2x - 7}{1.4}$ for $x = 1.26$

$$\begin{aligned} \frac{4.2x-7}{1.4} &= \frac{4.2(1.26)-7}{1.4} \\ &= \frac{5.292-7}{1.4} \\ &= \frac{-1.708}{1.4} \\ &= -1.22 \end{aligned}$$

$$\begin{aligned} \frac{4.2x-7}{1.4} &= \frac{4.2x}{1.4} + \frac{-7}{1.4} \\ &= 3x + (-5) \\ 3x - 5 &= 3(1.26) + (-5) \\ &= -1.22 \end{aligned}$$

c. Which form—simplified or not simplified—did you prefer to evaluate? Why?

Answers will vary.

Be prepared to share your solutions and methods.

6

Follow Up

Assignment

Use the Assignment for Lesson 6.2 in the Student Assignments book. See the Teacher's Resources and Assessments book for answers.

Skills Practice

Refer to the Skills Practice worksheet for Lesson 6.2 in the Student Assignments book for additional resources. See the Teacher's Resources and Assessments book for answers.

Assessment

See the Assessments provided in the Teacher's Resources and Assessments book for Chapter 6.

Check for Students' Understanding

A student submitted the following quiz. Grade the paper by marking each item with a √ or an **X**. Correct any mistakes.

Name Alicia Smith

Distributive Property Quiz

1. $2(x + 5) = 2x + 10$
   √

2. $2(3x - 6) = 6x - 6$
   X $6x - 12$

3. $-3x(4y - 10) = -12xy + 30$
   X $-12xy + 30x$

4. $5x(3x + 2y) = 15x + 10xy$
   X $15x^2 + 10xy$

5. $\frac{15x + 10}{5} = 3x + 2$
   √

6. $\frac{8x - 4}{4} = 2x + 1$
   X $2x - 1$

7. $12x + 4 = 3(4x + 1)$
   X $4(3x + 1)$

8. $-2x + 8 = -2(x - 4)$
   √

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6.3 Reverse Distribution

Factoring Algebraic Expressions

Learning Goals

In this lesson, you will:

- Use the distributive properties to factor expressions.
- Combine like terms to simplify expressions.

Key Terms

- factor
- common factor
- greatest common factor (GCF)
- coefficient
- like terms
- combining like terms

Essential Ideas

- The Distributive Properties are used to factor expressions.
- To factor an expression means to rewrite the expression as a product of factors.
- A common factor is a number or an algebraic expression that is a factor of two or more numbers or algebraic expressions.
- The greatest common factor is the largest factor that two or more numbers or terms have in common.
- A coefficient is the number that is multiplied by a variable in an algebraic expression.
- Terms are considered like terms if their variable portions are the same. Like terms can be combined.

Common Core State Standards for Mathematics

7.EE Expressions and Equations

Use properties of operations to generate equivalent expressions.

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

Overview

Algebraic expressions are factored using a distributive property. Expressions are rewritten by factoring out the GCF, or Greatest Common Factor. Expressions are simplified by combing like terms. Students will factor and evaluate expressions.

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Warm Up

1. What is the sum of 5 + 6?

   The sum of 5 + 6 is 11.

2. What is another way to write $5x$ using addition?

   $5x = x + x + x + x + x$

3. What is another way to write 6x using addition?

   $6x = x + x + x + x + x + x$

4. What is another way to write $5x + 6x$ using addition?

   $5x + 6x = x + x + x + x + x + x + x + x + x + x + x$

5. What is the sum of $5x + 6x$?

   The sum of $5x + 6x$ is $11x$.

6

6.3 REVERSE DISTRIBUTION

Factoring Algebraic Expressions

Learning Goals

In this lesson, you will:

- Use the distributive properties to factor expressions.
- Combine like terms to simplify expressions.

Key Terms

- factor
- common factor
- greatest common factor (GCF)
- coefficient
- like terms
- combining like terms

Many beginning drivers have difficulty with driving in reverse. They think that they must turn the wheel in the opposite direction of where they want the back end to go. But actually, the *reverse*, is true. To turn the back end of the car to the left, turn the steering wheel to the left. To turn the back end to the right, turn the wheel to the right.

Even after mastering reversing, most people would have difficulty driving that way all the time. But not Rajagopal Kawendar. In 2010, Kawendar set a world record for driving in reverse—over 600 miles at about 40 miles per hour!

Problem 1

Students use the Distributive Property in reverse to factor expressions. The greatest common factor is defined and students practice factoring out the GCF.

Grouping

Ask a student to read the introduction to Problem 1 aloud. Complete Question 1 then discuss the worked example as a class.

Discuss Phase, Question 1

- What does it mean to factor an algebraic expression?
- What is the product of $7(26) + 7(14)$?
- What is the product of $7(26 + 14)$ or $7(40)$?
- Which problem was easier to solve?
- Explain the types of problems where factoring helps with mental math?
- Is it easier to perform the multiplication and then subtract the products or subtract the numbers in the parenthesis and then perform the multiplication? Explain.

Problem 1 Factoring

You can use the Distributive Property in reverse. Consider the expression:

$$7(26) + 7(14)$$

Since both 26 and 14 are being multiplied by the same number, 7, the Distributive Property says you can add the multiplicands together first, and then multiply their sum by 7 just once.

$$7(26) + 7(14) = 7(26 + 14)$$

You have *factored* the original expression. To **factor** an expression means to rewrite the expression as a product of factors.

The number 7 is a *common factor* of both 7(26) and 7(14). A **common factor** is a number or an algebraic expression that is a factor of two or more numbers or algebraic expressions.

1. Factor each expression using a Distributive Property.

a. $4(33) - 4(28)$

$4(33 - 28)$

b. $16(17) + 16(13)$

$16(17 + 13)$

The Distributive Properties can also be used in reverse to factor algebraic expressions. For example, the expression $3x + 15$ can be written as $3(x) + 3(5)$, or $3(x + 5)$. The factor, 3, is the *greatest common factor* to both terms. The **greatest common factor** (GCF) is the largest factor that two or more numbers or terms have in common.

When factoring algebraic expressions, you can factor out the greatest common factor from all the terms.

> Consider the expression $12x + 42$. The greatest common factor of $12x$ and 42 is 6. Therefore, you can rewrite the expression as $6(2x + 7)$.

It is important to pay attention to negative numbers. When factoring an expression that contains a negative leading *coefficient*, or first term, it is preferred to factor out the negative sign. A **coefficient** is the number that is multiplied by a variable in an algebraic expression.

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Grouping

- Ask students to read the worked example on their own.
- Have students complete Question 2 with a partner. Then share the responses as a class.

Share Phase, Question 2

- Can the GCF contain variables?
- When should you factor out the negative sign?
- Is the coefficient of the first term in the expression negative or positive?
- How do you factor out a negative sign if the second term is positive?
- How can you check that you have factored the algebraic expression correctly?

Look at the expression $-2x + 8$. You can think about the greatest common factor as being the coefficient of -2.

$$-2x + 8 = (-2)x + (-2)(-4)$$
$$= -2(x - 4)$$

2. Rewrite each expression by factoring out the greatest common factor.

a. $7x + 14$
$7(x) + 7(2) = 7(x + 2)$

b. $9x - 27$
$9(x) - 9(3) = 9(x - 3)$

c. $10y - 25$
$5(2y) - 5(5) = 5(2y - 5)$

d. $8n + 28$
$4(2n) + 4(7) = 4(2n + 7)$

e. $3x^2 - 21x$
$3x(x) - 3x(7) = 3x(x - 7)$

f. $24a^2 + 18a$
$6a(4a) + 6a(3) = 6a(4a + 3)$

g. $15mn - 35n$
$5n(3m) - 5n(7) = 5n(3m - 7)$

h. $-3x - 27$
$-3(x) - 3(9) = -3(x + 9)$

i. $-6x + 30$
$-6(x) - 6(-5) = -6(x - 5)$

Problem 2

Students use the Distributive Property of Multiplication over Addition and Subtraction to combine like terms. They will develop the understanding that the coefficients of like terms are added, but the like variable remains as is.

Grouping

- Ask a student to read the introduction to Problem 2 aloud. Discuss the information and complete Questions 1 through 4 as a class.
- Have students complete Question 5 with a partner. Then share the responses as a class.

Discuss Phase, Questions 1 through 4

- Is a variable x a factor of each term?
- What does it mean to say that terms are "like terms"?
- Explain how to use the distributive property to combine like terms.
- Can combining like terms involve subtraction as well as addition? Explain.
- Why are $5ab$ and $22ab$ considered 'like terms'?
- Why are $32x^2$ and $44x^2$ considered 'like terms'?

Share Phase, Question 5

- How can you tell if an expression is already simplified?

Problem 2 Using the Distributive Properties to Simplify

So far, the Distributive Properties have provided ways to rewrite given algebraic expressions in equivalent forms. You can also use the Distributive Properties to simplify algebraic expressions.

Consider the algebraic expression $5x + 11x$.

1. What factors do the terms have in common?
   x

2. Rewrite $5x + 11x$ using the Distributive Property.
   $x(5 + 11)$

3. Simplify your expression.
   $16x$

The terms $5x$ and $11x$ are called **like terms**, meaning that their variable portions are the same. When you add $5x$ and $11x$ together, you are **combining like terms**.

4. Simplify each expression by combining like terms.
   a. $5ab + 22ab$
      $ab(5 + 22) = 27ab$
   b. $32x^2 - 44x^2$
      $x^2(32 - 44) = -12x^2$

5. Simplify each algebraic expression by combining like terms. If the expression is already simplified, state how you know.
   a. $6x + 9x$
      $15x$
   b. $-13y - 34y$
      $-47y$
   c. $14mn - 19mn$
      $-5mn$
   d. $8mn - 5m$
      The expression is already simplified. The terms do not contain the same variable portions.
   e. $6x^2 + 12x^2 - 7x^2$
      $11x^2$
   f. $6x^2 + 12x^2 - 7x$
      $18x^2 - 7x$
   g. $-3z - 8z - 7$
      $5z + 7$
   h. $5x + 5y$
      The expression is already simplified. The terms do not contain the same variable portions.

- There is a common phrase, "you cannot add apples to oranges". Explain how this phrase can be applied to combining like terms.
- Are $6x$ and $9x$ considered like terms? Why or why not?
- Are $8mn$ and $5m$ considered like terms? Why or why not?
- Are $6x$ and $9x$ considered like terms? Why or why not?
- Are $6x^2$, $12x^2$, and $-7x$ considered like terms? Why or why not?
- Are $-3z$, $-8z$, and -7 considered like terms? Why or why not?
- Are $5x$ and $5y$ considered like terms? Why or why not?

Problem 3

Students factor and evaluate expressions.

Grouping

Have students complete Questions 1 and 2 with a partner. Then share the responses as a class.

Share Phase, Questions 1 and 2

- Are $-24x$ and $16y$ considered like terms? Why or why not?
- Do the terms $-24x$ and $16y$ have a common factor? If so, what is it?
- What is the greatest common factor of the terms $-24x$ and $16y$?
- What distributive property was used to factor the expression?
- What is the decimal equivalent for $2\frac{1}{2}$?
- Is it easier to evaluate the expression using $2\frac{1}{2}$ or 2.5? Why?

Problem 3 More Factoring and Evaluating

1. Factor each expression.

a. $-24x + 16y = -8(3x + 2y)$

b. $-4.4 - 1.21z = -1.1(4 + 1.1z)$

c. $-27x - 33 = -3(9x + 11)$

d. $-2x - 9y = -1(2x + 9y)$ or $-(2x + 9y)$

e. $4x + (-5xy) - 3x = x(4 + (-5y) + (-3)) = x(1 - 5y)$

2. Evaluate each expression for the given value. Then factor the expression and evaluate the factored expression for the given value.

a. $-4x + 16$ for $x = 2\frac{1}{2}$

$$\begin{aligned} -4x + 16 &= -4\left(2\tfrac{1}{2}\right) + 16 \\ &= -\frac{\cancel{4}^{\,2}}{1}\left(\frac{5}{\cancel{2}_{\,1}}\right) + 16 \\ &= -10 + 16 \\ &= 6 \end{aligned}$$

$$\begin{aligned} -4x + 16 &= -4\,(x + (-4)) \\ &= -4\left(2\tfrac{1}{2} + (-4)\right) \\ &= -4\left(-1\tfrac{1}{2}\right) \\ &= -\frac{\cancel{4}^{\,2}}{1}\left(-\frac{3}{\cancel{2}_{\,1}}\right) \\ &= 6 \end{aligned}$$

b. $30x - 140$ for $x = 5.63$

$$\begin{aligned} 30x - 140 &= 30(5.63) - 140 \\ &= 168.9 - 140 \\ &= 28.9 \end{aligned}$$

$$\begin{aligned} 30x - 140 &= 10(3x - 14) \\ &= 10(3(5.63) - 14) \\ &= 10(16.89 - 14) \\ &= 10(2.89) \\ &\quad 28.9 \end{aligned}$$

c. Which form—simplified or not simplified—did you prefer to evaluate? Why?

Answers will vary.

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Problem 4

Students simplify each expression by combining like terms and evaluate expressions for the given value.

Grouping

Have students complete Questions 1 and 2 with a partner. Then share the responses as a class.

Share Phase, Questions 1 and 2

- Which terms in the expression are considered like terms?
- Which terms in the expression can be combined?
- What is the first step?
- What is the second step?
- If the mixed numbers are changed into decimals, will the answer be exact? Explain.

Problem 4 Combining Like Terms and Evaluating

1. Simplify each expression by combining like terms.

a. $30x - 140 - 23x = 7x - 140$

b. $-5(-2x - 13) - 7x = 10x + 65 + (-7x)$
$= 3x + 65$

c. $-4x - 5(2x - y) - 3y = -4x + (-10x) + 5y + (-3y)$
$= -14x + 2y$

d. $7.6x - 3.2(3.1x - 2.4) = 7.6x - 9.92x + 7.68$
$= -2.32x + 7.68$

e. $3\frac{2}{3}x - 1\frac{3}{4}\left(4x - 2\frac{1}{7}\right) = 3\frac{2}{3}x + \left(-1\frac{3}{4}\right)(4x) + \left(-1\frac{3}{4}\right)\left(-2\frac{1}{7}\right)$

$= 3\frac{2}{3}x + \left(-\frac{7}{\cancel{4}_{1}}\right)\left(\frac{\cancel{4}^{1}}{1}x\right) + \left(-\frac{\cancel{7}^{1}}{4}\right)\left(-\frac{15}{\cancel{7}_{1}}\right)$

$= 3\frac{2}{3}x + (-7x) + \frac{15}{4}$

$= \frac{11}{3}x + \left(-\frac{21}{3}\right)x + \frac{15}{4}$

$= -\frac{11}{3}x + \frac{15}{4}$

2. Evaluate each expression for the given value. Then combine the like terms in each expression and evaluate the simplified expression for the given value.

a. $-5x - 12 + 3x$ for $x = 2.4$

$-5x - 12 + 3x = -5(2.4) - 12 + 3(2.4)$
$= -12 + (-12) + 7.2$
$= -16.8$

$-5x - 12 + 3x = -2x - 12$
$= -2(2.4) - 12$
$= -4.8 - 12$
$= -16.8$

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b. $-2\frac{1}{2}x - 1\frac{2}{3}\left(6x + 2\frac{2}{5}\right)$ for $x = -1\frac{1}{4}$

$$-2\tfrac{1}{2}x - 1\tfrac{2}{3}\left(6x + 2\tfrac{2}{5}\right) = -2\tfrac{1}{2}\left(-1\tfrac{1}{4}\right) - 1\tfrac{2}{3}\left(6\left(-1\tfrac{1}{4}\right) + 2\tfrac{2}{5}\right)$$

$$= \left(-\tfrac{5}{2}\right)\left(-\tfrac{5}{4}\right) + \left(-\tfrac{5}{3}\right)\left(\tfrac{6}{1}\left(-\tfrac{5}{4}\right) + 2\tfrac{2}{5}\right)$$

$$= \tfrac{25}{8} + \left(-\tfrac{5}{3}\right)\left(\tfrac{\cancel{6}^{3}}{1}\left(-\tfrac{5}{\cancel{4}_{2}}\right) + 2\tfrac{2}{5}\right)$$

$$= \tfrac{25}{8} + \left(-\tfrac{5}{3}\right)\left(-\tfrac{15}{2} + 2\tfrac{2}{5}\right)$$

$$= \tfrac{25}{8} + \left(-\tfrac{5}{3}\right)\left(-\tfrac{75}{10} + \tfrac{24}{10}\right)$$

$$= \tfrac{25}{8} + \left(-\tfrac{\cancel{5}^{1}}{\cancel{3}_{1}}\right)\left(-\tfrac{\cancel{51}^{17}}{\cancel{10}_{2}}\right)$$

$$= \tfrac{25}{8} + \tfrac{17}{2}$$

$$= \tfrac{25}{8} + \tfrac{68}{8}$$

$$= \tfrac{93}{8} = 11\tfrac{5}{8}$$

$$-2\tfrac{1}{2}x - 1\tfrac{2}{3}\left(6x + 2\tfrac{2}{5}\right) = -2\tfrac{1}{2}x + \left(-1\tfrac{2}{3}\right)(6x) + \left(-1\tfrac{2}{3}\right)\left(2\tfrac{2}{5}\right)$$

$$= \left(-\tfrac{5}{2}\right)x + \left(-\tfrac{5}{3}\right)\left(\tfrac{\cancel{6}^{2}}{\cancel{1}_{1}}\right)x + \left(\tfrac{\cancel{5}^{1}}{\cancel{3}_{1}}\right)\left(\tfrac{\cancel{12}^{4}}{\cancel{5}_{1}}\right)$$

$$= \left(-\tfrac{5}{2}\right)x + (-10x) + (-4)$$

$$= \left(-\tfrac{25}{2}\right)x + (-4)$$

$$= \left(-\tfrac{25}{2}\right)\left(-1\tfrac{1}{4}\right) + (-4)$$

$$= \left(-\tfrac{25}{2}\right)\left(-\tfrac{5}{4}\right) + (-4)$$

$$= \tfrac{125}{8} + -\tfrac{32}{8}$$

$$= \tfrac{93}{8} = 11\tfrac{5}{8}$$

Be prepared to share your solutions and methods.

6

Follow Up

Assignment

Use the Assignment for Lesson 6.3 in the Student Assignments book. See the Teacher's Resources and Assessments book for answers.

Skills Practice

Refer to the Skills Practice worksheet for Lesson 6.3 in the Student Assignments book for additional resources. See the Teacher's Resources and Assessments book for answers.

Assessment

See the Assessments provided in the Teacher's Resources and Assessments book for Chapter 6.

Check for Students' Understanding

Match each example to the most appropriate term.

Example	Operation/Property
1. $12(3) + 12(2) = 12(3 + 2)$ (G)	A. Distributive Property of Multiplication over Subtraction
2. The 5 in $5x - 30$ (F)	B. common factor
3. $\frac{8 - 6}{3} = \frac{8}{3} - \frac{6}{3}$ (H)	C. combining like terms
4. $2(12 - 5) = 2(12) - 2(5)$ (A)	D. Distributive Property of Multiplication over Addition
5. $13y$ and $22y$ (I)	E. Distributive Property of Division over Addition
6. $10(5 + 2) = 10(5) + 10(2)$ (D)	F. Greatest Common Factor
7. $32x + 41x = 73x$ (C)	G. factoring
8. $\frac{8 + 6}{3} = \frac{8}{3} + \frac{6}{3}$ (E)	H. Distributive Property of Division over Subtraction
9. The 8 in $8(4) + 8(9)$ (B)	I. like terms

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6.4 Are They the Same or Different?

Verifying That Expressions Are Equivalent

Learning Goals

In this lesson, you will:

- Simplify algebraic expressions.
- Verify that algebraic expressions are equivalent by graphing, simplifying, and evaluating expressions.

Essential Ideas

- Expressions are equivalent if one expression can be transformed into the other.
- Expressions are equivalent if they have the same graph.

Common Core State Standards for Mathematics

7.EE Expressions and Equations

Use properties of operations to generate equivalent expressions.

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

Overview

Students explore two different methods to see if expressions in an equation are equivalent. Next, they will determine whether the expressions are equivalent using a specified method. Finally, students verify whether the expressions in equations are equivalent using the method of their choice.

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Warm Up

Consider the two expressions.

$2x + 6$

$x + 11$

1. If these two expressions are equal, what does that mean?

   If these two expressions are equal, that means they have the same value.

2. For what value of x does $2x + 6 = x + 11$?

   $2x + 6 = x + 11$, when x has a value of 5.

3. For what value of x does $2x + 6 \neq x + 11$?

   Answers will vary.

   $2x + 6 \neq x + 11$, when x has a value of 1.

4. If expressions are equivalent, what does that mean?

   If two expressions are equivalent, that means one expression can be transformed into the other so the expressions are essentially the same expression.

5. Are these two expressions equivalent?

   These two expressions are not equivalent because one expression cannot be transformed into the other. They are different expressions.

6. Can two expressions be equal but not equivalent? Explain your reasoning.

   Yes. Two expressions can be equal to each other for a specific value, but that does not mean the expressions are equivalent to each other for all values.

7. Can two expressions be equivalent but not equal? Explain your reasoning.

   No. If two expressions are equivalent then they are equal to each other for all values because they are essentially the same expression.

6

6.4 ARE THEY THE SAME OR DIFFERENT?

Verifying That Expressions Are Equivalent

Learning Goals

In this lesson, you will:

- Simplify algebraic expressions.
- Verify that algebraic expressions are equivalent by graphing, simplifying, and evaluating expressions.

Bart and Lisa are competing to see who can get the highest grades. But they are in different classes. In the first week, Lisa took a quiz and got 9 out of 10 correct for a 90%. Bart took a test and got 70 out of 100 for a 70%. Looks like Lisa won the first week!

The next week, Lisa took a test and got 35 out of 100 correct for a 35%. Bart took a quiz and got 2 out of 10 correct for a 20%. Lisa won the second week also!

Over the two weeks, it looks like Lisa was the winner. But look at the total number of questions and the total each of them got correct: Lisa answered a total of 110 questions and got a total of 34 correct for about a 31%. Bart answered a total of 110 questions and got a total of 72 correct for a 65%! Is Bart the real winner?

This surprising result is known as Simpson's Paradox. Can you see how it works?

Problem 1

Students examine work that contains expressions that are equal but not equivalent. Students will conclude that expressions are equivalent only if one expression can be transformed into the other or their graphs are the same.

Grouping

Have students complete Questions 1 through 9 with a partner. Then share the responses as a class.

Share Phase, Questions 1 through 3

- What is the expression on the left side of the equation equal to?
- What is the expression on the right side of the equation equal to?
- Is the expression on the left side of the equation equal to the expression on the right side of the equation?
- Is there a value for x that would make the expression on the left side of the equation equal to the expression on the right side of the equation? If so, what is it?
- If the expression on the left side of the equation is not equal to the expression on the right side of the equation, are the expressions considered equivalent?

Problem 1 Are They Equivalent?

Consider this equation:

$$-4(3.2x - 5.4) = 12.8x + 21.6$$

Keegan says that to tell if the expressions in an equation are equivalent (not just equal), you just need to evaluate each expression for the same value of x.

1. Evaluate the expression on each side of the equals sign for $x = 2$.

   $-4(3.2(2) - 5.4) = -4(6.4 - 5.4) = -4(1) = -4$

   $12.8(2) + 21.6 = 25.6 + 21.6 = 47.2$

2. Are these expressions equivalent? Explain your reasoning.

   No, since the left side is equal to -4, and the right side is equal to 47.2, these expressions are not equivalent.

Jasmine

Keegan's method can prove that two expressions are not equivalent, but it can't prove that they are equivalent.

3. Explain why Jasmine is correct. Provide an example of two expressions that verify your answer.

   There are expressions that might be equal but not equivalent.
   For example,

   $3x$ for $x = 2$ $\quad 3(2) = 6$

   $x + 4$ for $x = 2$ $\quad 2 + 4 = 6$

Kaitlyn

There is a way to prove that two expressions are equivalent, not just equal: use properties to try and turn one expression algebraically into the other.

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Share Phase, Questions 4 through 7

- What bounds were used for the x-axis to graph the equation?
- What bounds were used for the y-axis to graph the equation?
- Do both expressions have the same graph?
- When the two expressions have different graphs, what does this imply?
- What is the significance of the point at which the two graphs intersect?

4. Reconsider again the equation $-4(3.2x - 5.4) = 12.8x + 21.6$. Use the distributive properties to simplify the left side and to factor the right side to try to determine if these expressions are equivalent.

$-4(3.2x - 5.4) = (-4)(3.2x) + (-4)(-5.4) = -12.8x + 21.6$

$12.8x + 21.6 = (4)(3.2x) + (4)(-5.4) = 4(3.2x - 5.4)$

5. Are the expressions equivalent? Explain your reasoning.

No, they are not, because neither can be transformed into the other.

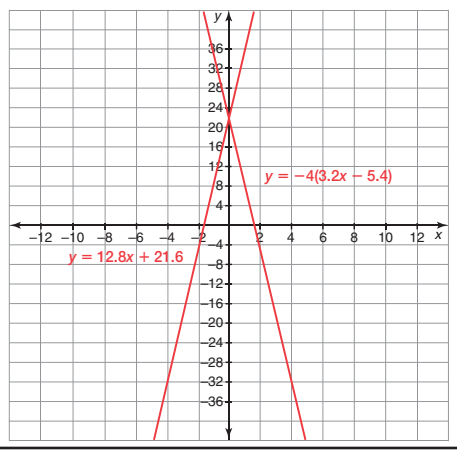
6. Will Kaitlyn's method always work? Explain your reasoning.

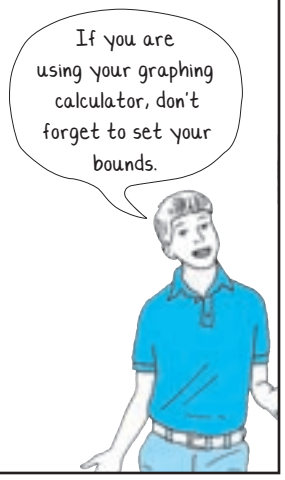
Yes, if one expression can be transformed into the other, then they must be equivalent. If not, they cannot be equivalent.

Jason

There is another method to verify the equivalence of expressions: graph them, and if their graphs are the same, then the expressions must be equivalent.

7. Graph each expression on your graphics calculator or other graphing technology. Sketch the graphs.





Problem 2

Students determine if two expressions are equivalent by evaluating the expressions. They will then determine if two expressions are equivalent by graphing the expressions.

Grouping

Have students complete Questions 1 through 3 with a partner. Then share the responses as a class.

Share Phase, Questions 1 and 2

- What is the expression on the left side of the equation equal to?
- What is the expression on the right side of the equation equal to?
- Is the expression on the left side of the equation equal to the expression on the right side of the equation?

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8. Are the expressions equivalent? Explain your reasoning.

Clearly the graphs are not the same and therefore the expressions are not equivalent.

9. Which method is more efficient for this problem? Explain your reasoning.

Answers will vary, but evaluating for a specific value is the most efficient in this case.

Problem 2 Are These Equivalent?

Determine whether the expressions are equivalent using the specified method.

1. $3.1(-2.3x - 8.4) + 3.5x = -3.63x + (-26.04)$ by evaluating for $x = 1$.

$$\begin{aligned} 3.1(-2.3x - 8.4) + 3.5x &= 3.1(-2.3(1) - 8.4) + 3.5(1) \\ &= 3.1(-2.3 - 8.4) + 3.5 \\ &= 3.1(-10.7) + 3.5 \\ &= -33.17 + 3.5 = -29.67 \\ -3.63x + (-26.04) &= -3.63(1) + (-26.04) = -29.67 \end{aligned}$$

These are equivalent.

2. $3\frac{1}{3}\left(-3x - 2\frac{1}{10}\right) = 4\frac{3}{4} = -2\frac{1}{2}\left(4x + \frac{2}{5}\right) - 1\frac{1}{4}$ by simplifying each side.

$$3\frac{1}{3}\left(-3x - 2\frac{1}{10}\right) + 4\frac{3}{4} = \left(\frac{10}{\cancel{3}}\right)\left(-\frac{\cancel{3}}{1}x\right) + \left(\frac{\cancel{10}}{\cancel{3}}\right)\left(-\frac{\cancel{21}}{\cancel{10}}\right) + 4\frac{3}{4}$$

$$= -10x + (-7) + 4\frac{3}{4}$$

$$= -10x + \left(-2\frac{1}{4}\right)$$

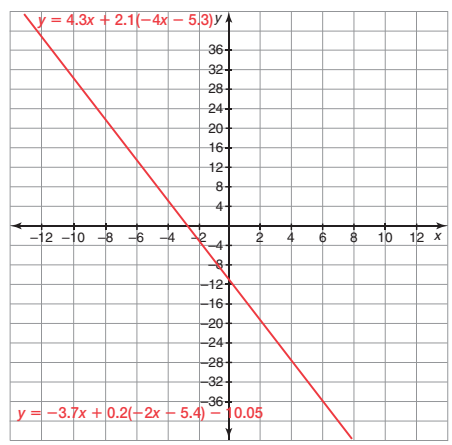
$$-2\frac{1}{2}\left(4x + \frac{2}{5}\right) - 1\frac{1}{4} = \left(-\frac{5}{\cancel{2}}\right)\left(\frac{\cancel{4}}{1}x\right) + \left(-\frac{\cancel{5}}{\cancel{2}}\right)\left(\frac{\cancel{2}}{\cancel{5}}\right) + \left(-1\frac{1}{4}\right)$$

$$= -10x + \left(-2\frac{1}{4}\right)$$

Yes, they are equivalent.

3. Graph each expression using graphing technology to determine if these are equivalent expressions. Sketch the graphs.

$4.3x + 2.1(-4x - 5.3) = -3.7x + 0.2(-2x - 5.4) - 10.05$



Yes, they are equivalent.

Share Phase, Question 3

- What bounds were used for the *x*-axis to graph the equation?
- What bounds were used for the *y*-axis to graph the equation?
- Do both expressions have the same graph?
- When the two expressions have the same graphs, what does this imply?

Problem 3

Students determine if two expressions are equivalent by evaluating the expressions. They will then determine if two expressions are equivalent by graphing the expressions.

Grouping

Have students complete Questions 1 through 3 with a partner. Then share the responses as a class.

Share Phase, Questions 1 and 2

- What is the expression on the left side of the equation equal to?
- What is the expression on the right side of the equation equal to?
- Is the expression on the left side of the equation equal to the expression on the right side of the equation?

Problem 3 What About These? Are They Equivalent?

For each equation, verify whether the expressions are equivalent using any of these methods. Identify the method and show your work.

1. $3\frac{1}{3}\left(6x - 2\frac{1}{5}\right) + 5\frac{1}{3} = -6\frac{2}{3}\left(-3x + 1\frac{6}{5}\right) + 12\frac{2}{3}$

$$3\frac{1}{3}\left(6x - 2\frac{1}{5}\right) + 5\frac{1}{3} = \frac{10}{\cancel{3}_{1}}\left(\frac{\cancel{6}^{2}}{1}x\right) + \frac{\cancel{10}^{2}}{3}\left(-\frac{11}{\cancel{5}_{1}}\right) + 5\frac{1}{3}$$

$$= 20x + \left(-\frac{22}{3}\right) + 5\frac{1}{3}$$

$$= 20x + (-2)$$

$$-6\frac{2}{3}\left(-3x + 1\frac{6}{5}\right) + 12\frac{2}{3} = \left(-\frac{20}{\cancel{3}_{1}}\right)\left(-\frac{\cancel{3}^{1}}{1}x\right) + \left(-\frac{\cancel{20}^{4}}{\cancel{3}_{1}}\right)\left(\frac{11}{5}\right) + 12\frac{2}{3}$$

$$= 20x + \left(-\frac{44}{3}\right) + 12\frac{2}{3}$$

$$= 20x + (-2)$$

They are equivalent since they simplify to the same expression.

2. $4.2(-3.2x - 8.2) - 7.6x = -15.02x + 3(4.1 - 3x)$

$$4.2(-3.2x - 8.2) - 7.6x = 4.2(-3.2(2) - 8.2) - 7.6(2)$$

$$= 4.2(-14.6) + (-15.2)$$

$$= -76.52$$

$$= -15.02x + 3(4.1 - 3x)$$

$$= -15.02(2) + 3(4.1 - 9(2))$$

$$= -30.04 + 3(-13.9)$$

$$= -71.74$$

These expressions are not equivalent.

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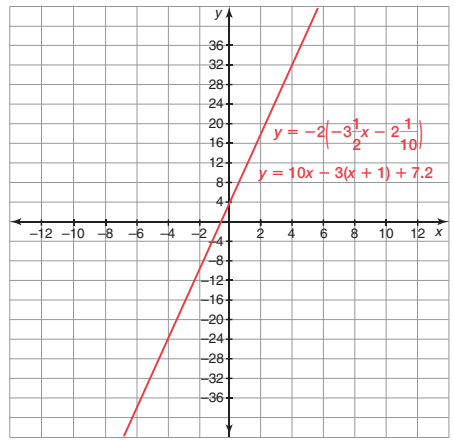
Share Phase, Question 3

- What bounds were used for the x-axis to graph the equation?
- What bounds were used for the y-axis to graph the equation?
- Do both expressions have the same graph?
- When the two expressions have the same graphs, what does this imply?

3. $-2\left(-3\frac{1}{2}x - 2\frac{1}{10}\right) = 10x - 3(x + 1) + 7.2$



These expressions are equivalent because they are represented by the same graph.

Be prepared to share your solutions and methods.

Follow Up

Assignment

Use the Assignment for Lesson 6.4 in the Student Assignments book. See the Teacher's Resources and Assessments book for answers.

Skills Practice

Refer to the Skills Practice worksheet for Lesson 6.4 in the Student Assignments book for additional resources. See the Teacher's Resources and Assessments book for answers.

Assessment

See the Assessments provided in the Teacher's Resources and Assessments book for Chapter 6.

Check for Students' Understanding

1. Write two expressions that are equal but not equivalent. Then, show that the two expressions are equal.

   $3x + 9$

   $x + 17$

   Let $x = 4$

   $3x + 9 = x + 17$

   $3(4) + 9 = 4 + 17$

   $21 = 21$

2. Write two expressions that are equivalent. Then, show that the two expressions are equal.

   $3x + 9$

   $3(x + 3)$

   $3x + 9 = 3(x + 9)$

   $3x + 9 = 3x + 9$

   The expressions are the same.

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6.5 It Is Time to Justify!

Simplifying Algebraic Expressions Using Operations and Their Properties

Learning Goal

In this lesson, you will:

- Simplify algebraic expressions using operations and their properties.

Essential Ideas

- Expressions are equivalent if one expression can be transformed into the other.
- Operations and properties are used to simplify algebraic expressions.

Common Core State Standards for Mathematics

7.EE Expressions and Equations

Use properties of operations to generate equivalent expressions.

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

Overview

To verify that two algebraic expressions are equivalent, one side of the equation is simplified to match the other side. Each step of the simplification process is justified by using an operation or property. An example is provided. Students complete tables by providing the justification for each given step of the solution. They will then complete tables by providing the steps that match each given justification. Last, they provide both the steps and the justifications for four different problems.

Note

This lesson might be beyond what you may want to do with students. Only you can decide what is most appropriate for your class. If you choose to use this lesson, pay special attention to students who provide alternate solution paths. The answers in the lesson only provide one solution path, but keep in mind that no one path is more correct than another. All valid solution paths should be recognized and verified.

Warm Up

Write an example of each operation or property.

Answers will vary for all questions.

Example	Operation/Property
1. $-4x + (-21x) + 9 + 6x + (-8) = -25x + 9 + 6x + (-8)$	Addition
2. $-25x + 6x + 9 + (-8) = -19x + 1$	Addition of like terms
3. $-5(x + 4.3) - 5(x + 4.3) = -10(x + 4.3)$	Factoring Distributive Property of Multiplication over Subtraction
4. $-4x + 159 + (-8x) = -4x + (-8x) + 159$	Commutative Property of Addition
5. $3(-x + 50) = (-3x) + 150$	Distributive Property of Multiplication over Addition

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6.5 IT IS TIME TO JUSTIFY!

Simplifying Algebraic Expressions Using Operations and Their Properties

Learning Goal

In this lesson, you will:

- Simplify algebraic expressions using operations and their properties.

Have you ever been asked to give reasons for something that you did? When did this occur? Were your reasons accepted or rejected? Is it always important to have reasons for doing something or believing something?

Problem 1

Given each step of the solution path, students supply each justification (operation or property) needed to show two expressions are equivalent. Then, when given each justification (operation or property) used in a solution path, students will supply each step needed to show two expressions are equivalent.

Grouping

Ask a student to read the introduction to Problem 1 aloud. Discuss the information and worked example as a class.

Problem 1 Justifying!!

One method for verifying that algebraic expressions are equivalent is to simplify the expressions into two identical expressions.

For this equation, the left side is simplified completely to show that the two expressions are equivalent.

$$-2\frac{1}{2}\left(1\frac{1}{3}x - \frac{2}{5}\right) + 2 = -3\frac{1}{3}x + 3$$

Step	Justification
$-2\frac{1}{2}\left(1\frac{1}{3}x - \frac{2}{5}\right) + 2 =$	Given
$\left(-\frac{5}{\cancel{2}_{1}}\right)\left(\frac{\cancel{4}^{2}}{3}x\right) + \left(-\frac{\cancel{5}^{1}}{\cancel{2}_{1}}\right)\left(-\frac{\cancel{2}^{1}}{\cancel{5}_{1}}\right) + 2 =$	Distributive Property of Multiplication over Addition
$-\frac{10}{3}x + 1 + 2 =$	Multiplication
$-3\frac{1}{3}x + 3$	Addition Yes, they are equivalent.

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Grouping

Have students complete Questions 1 and 2 with a partner. Then share the responses as a class.

Share Phase, Question 1

- When is the justification 'given' used?
- Compare the first step to the second step. What changed?
- Was an operation or property used to make this change? Which one?
- Compare the first second to the third step. What changed?
- Was an operation or property used to make this change? Which one?
- How do you know when you are on the last step?
- What should the last step look like?
- What is the difference between addition and addition of like terms? When should one be used instead of the other?
- How do you know what to write as the first step?

1. Use an operation or a property to justify each step and indicate if the expressions are equivalent.

a. $-4(-3x - 8) - 4x + 8 = -8x + 40$

Step	Justification
$-4(-3x - 8) - 4x + 8 =$	Given
$12x + 32 + (-4x) + 8 =$	Distributive Property of Multiplication over Addition
$12x + (-4x) + 32 + 8 =$	Commutative Property of Addition
$-8x + 40 =$	Addition of Like Terms Yes, they are equivalent.

b. $-2.1(-3.2x - 4) + 1.2(2x - 5) = 9.16x + 3.4$

Step	Justification
$-2.1(-3.2x - 4) + 1.2(2x - 5) =$	Given
$6.72x + 8.4 + 2.4x + (-6) =$	Distributive Property of Multiplication over Subtraction
$6.72x + 2.4x + 8.4 + (-6) =$	Commutative Property of Addition
$9.16x + 2.4 =$	Addition of Like Terms No, they are not equivalent.

c. $\frac{-4x - 9}{2} + \frac{-3x + 7}{3} = -3x + \left(-\frac{13}{6}\right)$

Step	Justification
$\frac{-4x - 9}{2} + \frac{-3x + 7}{3} =$	Given
$\frac{-4x}{2} + \left(-\frac{9}{2}\right) + \frac{-3x}{3} + \frac{7}{3} =$	Distributive Property of Multiplication over Addition
$-2x + \left(-\frac{9}{2}\right) + (-x) + \frac{7}{3} =$	Division
$-2x + (-x) + \left(-\frac{9}{2}\right) + \frac{7}{3} =$	Commutative Property of Addition
$-3x + \left(-\frac{27}{6}\right) + \frac{14}{6} =$	Addition of Like Terms
$-3x + \left(-\frac{13}{6}\right) =$	Addition of Like Terms Yes, they are equivalent.

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Share Phase, Question 2

- When is the justification `given' used?
- Compare the first step to the second step. What changed?
- Was an operation or property used to make this change? Which one?
- Compare the first second to the third step. What changed?
- Was an operation or property used to make this change? Which one?
- How do you know when you are on the last step?
- What should the last step look like?
- What is the difference between addition and addition of like terms? When should one be used instead of the other?
- How do you know what to write as the first step?

2. For each equation, simplify the left side completely using the given operation or property that justifies each step and indicate if the expressions are equivalent.

a. $-4x - \frac{6x - 7}{5} = -\frac{26}{5}x + \frac{7}{5}$

Step	Justification
$-4x - \frac{6x - 7}{5} =$	Given
$-4x + \left(-\frac{6x}{5}\right) - \left(-\frac{7}{5}\right) =$	Distributive Property of Division over Subtraction
$-\frac{20}{5}x + \left(-\frac{6}{5}x\right) + \frac{7}{5} =$	Division
$-\frac{26}{5}x + \frac{7}{5} =$	Addition of Like Terms Yes, they are equivalent.

b. $-4x - 3(3x - 6) + 8(2.5x + 3.5) = 7x + 46$

Step	Justification
$-4x - 3(3x - 6) + 8(2.5x + 3.5) = 7x + 46$	Given
$-4x + (-9x) + 18 + 20x + 28 =$	Distributive Property of Multiplication over Addition
$-13x + 18 + 20x + 28 =$	Addition
$-13x + 20x + 18 + 28 =$	Commutative Property of Addition
$7x + 46 =$	Addition of Like Terms Yes, they are equivalent.

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Problem 2

Students simplify one or both sides of an equation to determine if the two expressions are equal. They will justify each step with an operation or property.

Grouping

Have students complete Questions 1 through 4 with a partner. Then share the responses as a class.

Share Phase, Questions 1 and 2

- How do you begin? What do you write as the first step?
- What is the objective of the problem?
- If you can't think of a justification, what should you do?
- Can you skip steps? If so, when? How many?
- How many steps were used to solve the problem?
- Why do some problems have more steps than others? What causes this to happen?
- How many different changes can be made in one step?
- Can you use a justification more than one time in the same problem? How many times can you use the same justification in one problem?
- How do you know when you are on the last step?

Problem 2 Simplify and Justify!

For each equation, simplify the left side completely to determine if the two expressions are equivalent. Use an operation or a property to justify each step and indicate if the expressions are equivalent.

1. $-4x + 3(-7x + 3) - 2(-3x + 4) = -19x + 1$

Step	Justification
$-4x + 3(-7x + 3) - 2(-3x + 4) = -19x + 1$	Given
$-4x + (-21x) + 9 + 6x + (-8) =$	Distributive Property of Multiplication over Addition
$-25x + 9 + 6x + (-8) =$	Addition
$-25x + 6x + 9 + (-8) =$	Commutative Property of Addition
$-19x + 1 =$	Addition of Like Terms Yes, they are equivalent.

2. $-5(x + 4.3) - 5(x + 4.3) = -10x - 43$

Step	Justification
$-5(x + 4.3) - 5(x + 4.3) =$	Given
$-10(x + 4.3) =$ $-5x + (-21.5) + (-5x) + (-21.5) =$	Factoring Distributive Property of Mult. over Sub. Distributive Property of Multiplication over Addition
$-10x + (-43) =$ $-5x + (-5x) + (-21.5) + (-21.5) =$	Addition Commutative Property of Addition
$-10x - 43$	Addition of Like Terms Yes, they are equivalent.

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Share Phase, Questions 3 and 4

- How do you begin? What do you write as the first step?
- What is the objective of the problem?
- If you can't think of a justification, what should you do?
- Can you skip steps? If so, when? How many?
- How many steps were used to solve the problem?
- Why do some problems have more steps than others? What causes this to happen?
- How many different changes can be made in one step?
- Can you use a justification more than one time in the same problem? How many times can you use the same justification in one problem?
- How do you know when you are on the last step?

For each equation, simplify the left side and the right side completely to determine if the two expressions are equivalent. Use an operation or a property to justify each step and indicate if the expressions are equivalent.

3. $-3(-x + 17) + 7(-x + 30) - 8x = -6x + 3(-x + 50) + 9$

Left side

Step	Justification
$-3(-x + 17) + 7(-x + 30) - 8x =$	Given
$3x + (-51) + (-7x) + 210 + (-8x) =$	Distributive Property of Multiplication over Addition
$3x + (-7x) + (-51) + 210 + (-8x) =$	Commutative Property of Addition
$-4x + 159 + (-8x) =$	Addition
$-4x + (-8x) + 159 =$	Commutative Property of Addition
$-12x + 159 =$	Addition of Like Terms

Right side

Step	Justification
$= -6x + 3(-x + 50) + 9$	Given
$= -6x + (-3x) + 150 + 9$	Distributive Property of Multiplication over Addition
$= -9x + 159$	Addition of Like Terms No, they are not equivalent.

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4. $-3\frac{1}{2}(4x + 6) + 1\frac{1}{3}(-6x - 3) - 4x = -5\frac{1}{2}x - 13 + 2\frac{1}{2}(-6x + 1) - 5\frac{1}{2}x - 14\frac{1}{2}$

Left side

Step	Justification
$-3\frac{1}{2}(4x + 6) + 1\frac{1}{3}(-6x - 3) - 4x =$	Given
$\left(-\frac{7}{2}\right)(4x) + \left(-\frac{7}{2}\right)(6) + 1\frac{1}{3}(-6x - 3) + (-4x) =$	Distributive Property of Multiplication over Addition
$-14x + (-21) + 1\frac{1}{3}(-6x - 3) + (-4x) =$	Multiplication
$-14x + (-21) + \frac{4}{3}(-6x) + \frac{4}{3}(-3) + (-4x) =$	Distributive Property of Multiplication over Addition
$-14x + (-21) + (-8x) + (-4) + (-4x) =$	Multiplication
$-14x + (-8x) + (-21) + (-4) + (-4x) =$	Commutative Property of Addition
$-22x + (-25) + (-4x) =$	Addition of Like Terms
$-22x + (-4x) + (-25) =$	
$-26x + (-25) =$	Addition of Like Terms

Right side

Step	Justification
$= -5\frac{1}{2}x - 13 + 2\frac{1}{2}(-6x + 1) - 5\frac{1}{2}x - 14\frac{1}{2}$	Given
$= -5\frac{1}{2}x + (-13) + \left(\frac{5}{2}\right)(-6x) + \left(\frac{5}{2}\right)(1) + \left(-5\frac{1}{2}x\right) + \left(-14\frac{1}{2}\right)$	Distributive Property of Multiplication over Addition
$= -5\frac{1}{2}x + (-13) + (-15x) + \left(\frac{5}{2}\right) + \left(-5\frac{1}{2}x\right) + \left(-14\frac{1}{2}\right)$	Multiplication
$= -5\frac{1}{2}x + (-15x) + (-13) + \left(\frac{5}{2}\right) + \left(-5\frac{1}{2}x\right) + \left(-14\frac{1}{2}\right)$	Commutative Property of Addition
$= -20\frac{1}{2}x + \left(-10\frac{1}{2}\right) + \left(-5\frac{1}{2}x\right) + \left(-14\frac{1}{2}\right)$	Addition
$= -20\frac{1}{2}x + \left(-5\frac{1}{2}x\right) + \left(-10\frac{1}{2}\right) + \left(-14\frac{1}{2}\right)$	Commutative Property of Addition
$= -26x + (-25)$	Addition of Like Terms Yes, they are equivalent.

Be prepared to share your solutions and methods.

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Follow Up

Assignment

Use the Assignment for Lesson 6.5 in the Student Assignments book. See the Teacher's Resources and Assessments book for answers.

Skills Practice

Refer to the Skills Practice worksheet for Lesson 6.5 in the Student Assignments book for additional resources. See the Teacher's Resources and Assessments book for answers.

Assessment

See the Assessments provided in the Teacher's Resources and Assessments book for Chapter 6.

Check for Students' Understanding

Identify the operation or property that was used in each equation shown.

1. $3(-x + 50) = (-3x) + 150$

   Distributive Property of Multiplication over Addition

2. $-5(x + 4.3) - 5(x + 4.3) = -10(x + 4.3)$

   Factoring Distributive Property of Multiplication over Subtraction

3. $-25x + 6x + 9 + (-8) = -19x + 1$

   Addition of Like Terms

4. $-4x + 159 + (-8x) = -4x + (-8x) + 159$

   Commutative Property of Addition

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Chapter 6 Summary

Key Terms

- variable (6.1)
- algebraic expression (6.1)
- evaluate an algebraic expression (6.1)
- Distributive Property of Multiplication over Addition (6.2)
- Distributive Property of Multiplication over Subtraction (6.2)
- Distributive Property of Division over Addition (6.2)
- Distributive Property of Division over Subtraction (6.2)
- factor (6.3)
- common factor (6.3)
- greatest common factor (GCF) (6.3)
- coefficient (6.3)
- like terms (6.3)
- combining like terms (6.3)

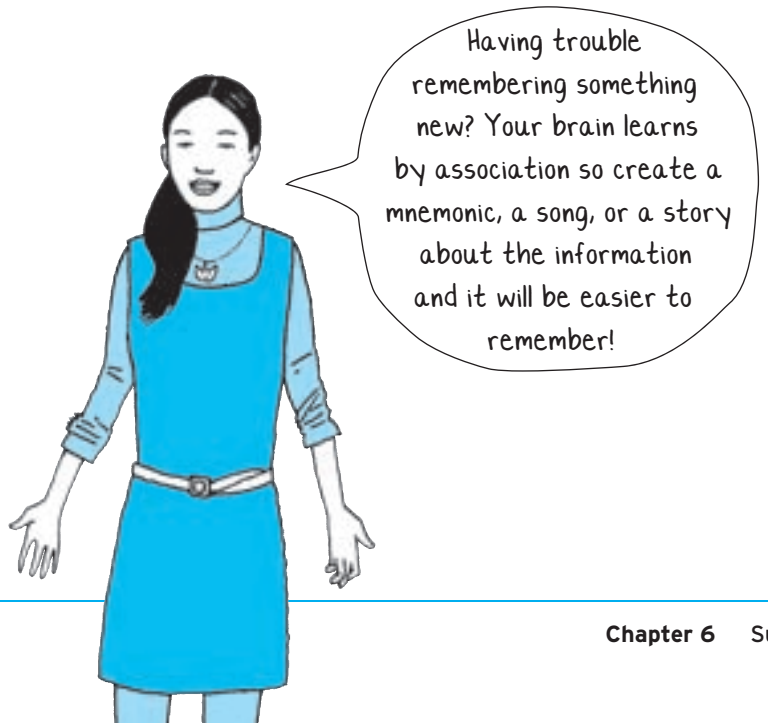
6.1 Writing Algebraic Expressions

When a mathematical process is repeated over and over, a mathematical phrase, called an algebraic expression, can be used to represent the situation. An algebraic expression is a mathematical phrase involving at least one variable and sometimes numbers and operation symbols.

Example

The algebraic expression $2.49p$ represents the cost of p pounds of apples.

One pound of apples costs 2.49(1), or \$2.49. Two pounds of apples costs 2.49(2), or \$4.98.



6.1 Evaluating Algebraic Expressions

To evaluate an algebraic expression, replace each variable in the expression with a number or numerical expression and then perform all possible mathematical operations.

Example

The expression $2x - 7$ has been evaluated for these values of x: 9, 2, -3, and 4.5.

$$2(9) - 7 = 18 - 7 = 11 \qquad 2(2) - 7 = 4 - 7 = -3 \qquad 2(-3) - 7 = -6 - 7 = -13 \qquad 2(4.5) - 7 = 9 - 7 = 2$$

6.2 Using the Distributive Property of Multiplication over Addition to Simplify Numerical Expressions

The Distributive Property of Multiplication over Addition states that if a, b, and c are any real numbers, then $a \bullet (b + c) = a \bullet b + a \bullet c$.

Example

A model is drawn and an expression written to show how the Distributive Property of Multiplication over Addition can be used to solve a multiplication problem.

6(820)

	800	20
6	4800	120

$$\begin{aligned} &6(800 + 20) \\ &= 6(800) + 6(20) \\ &= 4800 + 120 \\ &= 4920 \end{aligned}$$

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6.2 Using the Distributive Properties to Simplify and Evaluate Algebraic Expressions

Including the Distributive Property of Multiplication over Addition, there are a total of four different forms of the Distributive Property.

Another Distributive Property is the Distributive Property of Multiplication over Subtraction, which states that if a, b, and c are any real numbers, then $a \bullet (b - c) = a \bullet b - a \bullet c$.

The Distributive Property of Division over Addition states that if a, b, and c are real numbers and $c \neq 0$, then $\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$.

The Distributive Property of Division over Subtraction states that if a, b, and c are real numbers and $c \neq 0$, then $\frac{a - b}{c} = \frac{a}{c} - \frac{b}{c}$.

Example

The Distributive Properties have been used to simplify the algebraic expression. The simplified expression is then evaluated for $x = 2$.

$$\begin{aligned}\frac{4(6x - 7) + 10}{3} &= \frac{24x - 28 + 10}{3}\\ &= \frac{24x - 18}{3}\\ &= \frac{24x}{3} - \frac{18}{3}\\ &= 8x - 6\end{aligned}$$

$$\begin{aligned}8x - 6 &= 8(2) - 6\\ &= 16 - 6\\ &= 10\end{aligned}$$

6.3 Using the Distributive Properties to Factor Expressions

The Distributive Properties can be used in reverse to rewrite an expression as a product of factors. When factoring expressions, it is important to factor out the greatest common factor from all the terms. The greatest common factor (GCF) is the largest factor that two or more numbers or terms have in common.

Example

The expression has been rewritten by factoring out the greatest common factor.

$$\begin{aligned}24x^3 + 3x^2 - 9x &= 3x(8x^2) + 3x(x) - 3x(3)\\ &= 3x(8x^2 + x - 3)\end{aligned}$$

6.3 Combining Like Terms to Simplify Expressions

Like terms are terms whose variable portions are the same. When you add like terms together, you are combining like terms. You can combine like terms to simplify algebraic expressions to make them easier to evaluate.

Example

Like terms have been combined to simplify the algebraic expression. The simplified expression is then evaluated for $x = 5$.

$$\begin{aligned} 7x - 2(3x - 4) &= 7x - 6x + 8 \\ &= x + 8 \end{aligned} \qquad \begin{aligned} x + 8 &= 5 + 8 \\ &= 13 \end{aligned}$$

6.4 Determining If Expressions Are Equivalent by Evaluating

Evaluate the expression on each side of the equal sign for the same value of x. If the results are the same, then the expressions are equivalent.

Example

The expressions are equivalent.

$$(x + 12) + (4x - 9) = 5x + 3 \text{ for } x = 1$$

$$\begin{aligned} (1 + 12) + (4 \cdot 1 - 9) &\stackrel{?}{=} 5 \cdot 1 + 3 \\ 13 + (-5) &\stackrel{?}{=} 5 + 3 \\ 8 &= 8 \end{aligned}$$

6.4 Determining If Expressions Are Equivalent by Simplifying

The Distributive Properties and factoring can be used to determine if the expressions on each side of the equal sign are equivalent.

Example

The expressions are equivalent.

$$\begin{aligned} 5\left(-3x - \frac{2}{5}\right) &= -\frac{1}{3}(45x + 6) \\ -15x - 2 &= -15x - 2 \end{aligned}$$

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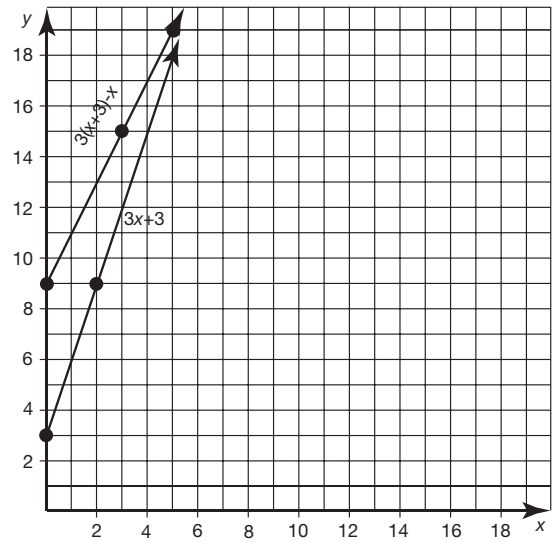
6.4 Determining If Two Expressions Are Equivalent by Graphing

Expressions can be graphed to determine if the expressions are equivalent. If the graph of each expression is the same, then the expressions are equal.

Example

$3(x + 3) - x = 3x + 3$ is not true because the graph of each expression is not the same.



6.5 Simplifying Algebraic Expressions Using Operations and Their Properties

Simplify each side completely to determine if the two expressions are equivalent. Use an operation or a property to justify each step and indicate if the expressions are equivalent.

Example

$-3(-2x - 9) - 3x - 7 = 3x + 20$

Step	Justification
$-3(-2x + (-9)) - 3x - 7 =$	Given
$6x + 27 + (-3x) + (-7) =$	Distributive Property of Multiplication over Addition
$6x + (-3x) + 27 + (-7) =$	Commutative Property of Addition
$3x + 20 =$	Addition of Like Terms Yes, they are equivalent.

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